TRIGONOMETRÍA

Profesor
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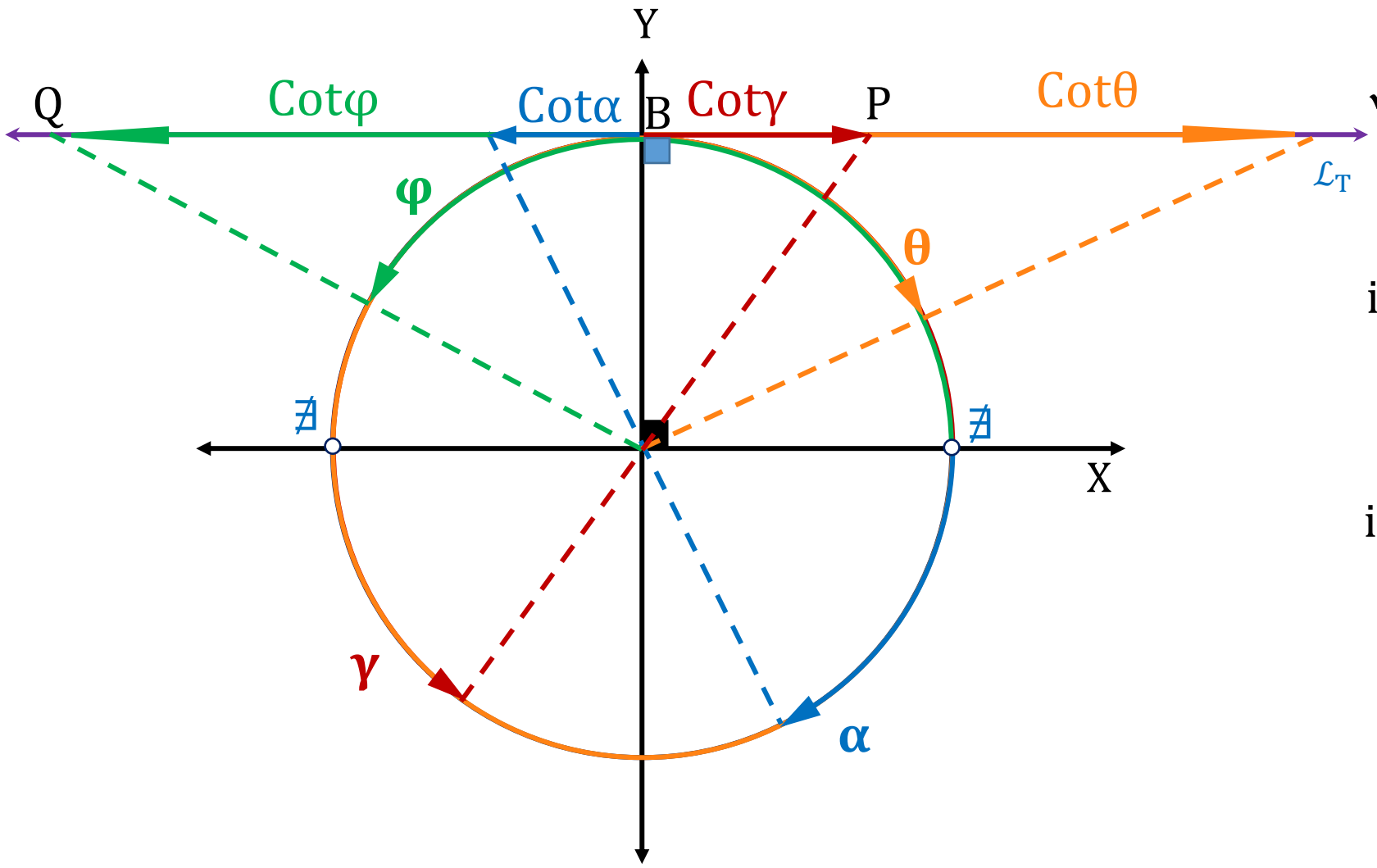
CIRCUNFERENCIA TRIGONOMÉTRICA

COTANGENTE, SECANTE, COSECANTE Y LÍNEAS AUXILIARES

4.4) Representación de la línea Cotangente:



✓ $\theta < \alpha < \varphi < \gamma$

✓ $\text{Cot}\varphi < \text{Cot}\alpha < \text{Cot}\gamma < \text{Cot}\theta$

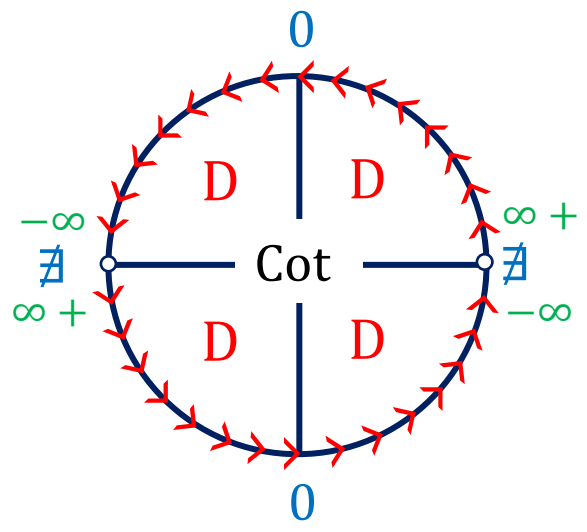
i. Valor $\begin{cases} (+): \text{Cot}\theta \\ (-): \text{Cot}\alpha \\ 0 \ : \text{Cot}\dfrac{3\pi}{2} \end{cases}$

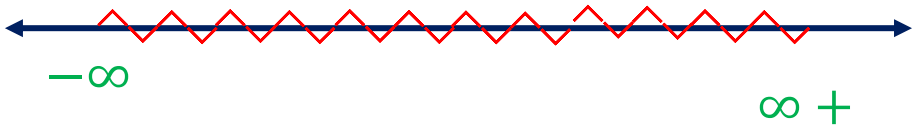
ii. Longitud: $(+)$, $|\text{Cotangente}|$

$\overline{BP} = |\text{Cot}\gamma| = \text{Cot}\gamma$

$\overline{BQ} = |\text{Cot}\varphi| = -\text{Cot}\varphi$

4.4.1)Variación analítica :





i. Extensión:

$$\forall\, \theta \in \mathbb{R} - \{n\pi; n \in \mathbb{Z}\}$$

$$-\infty < \text{Cot}\theta < \infty +$$

- Si: $\theta \in$ IC

  $\mathbf{0 < Cot\theta}$

- Si: $\theta \in$ IIC

  $\mathbf{Cot\theta < 0}$

- Si: $\theta \in$ IIIC

  $\mathbf{0 < Cot\theta}$

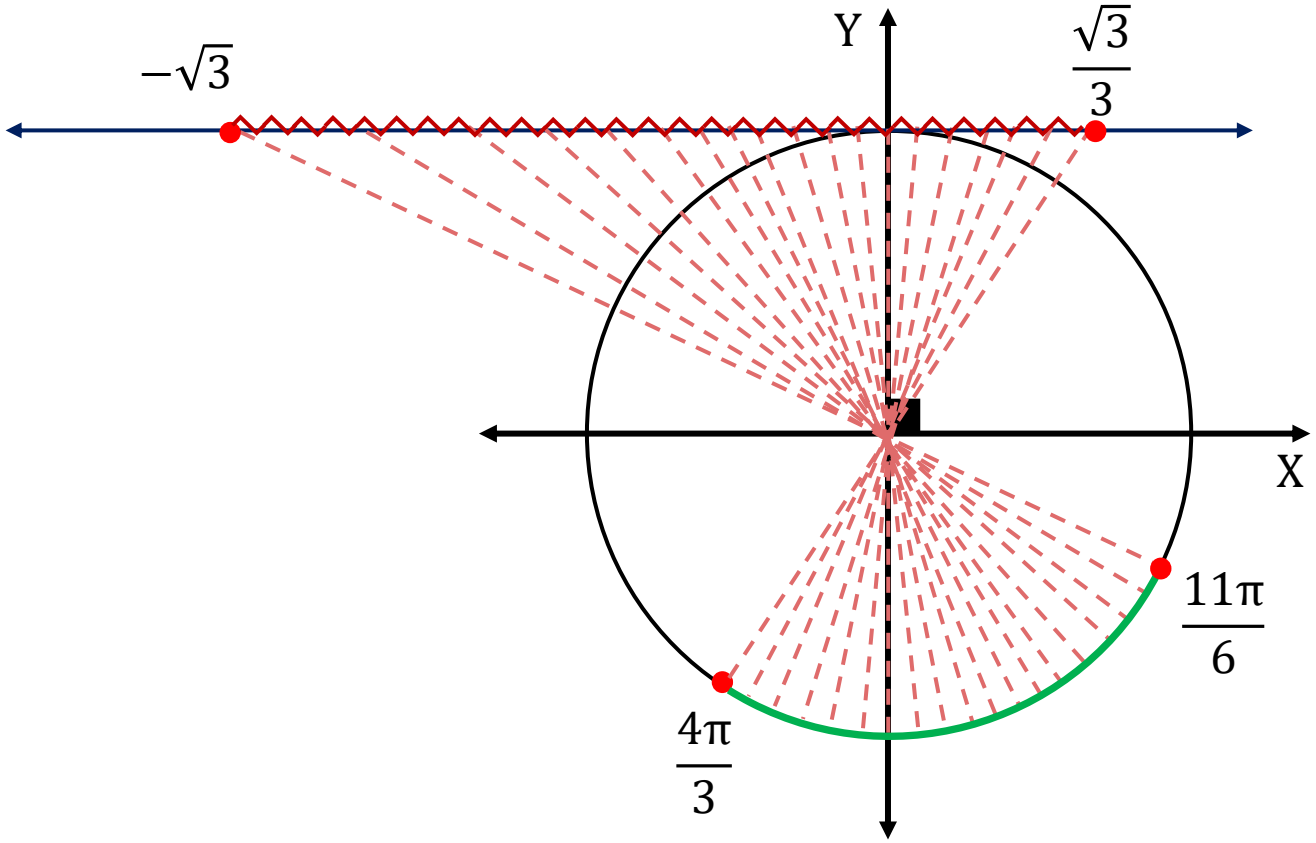
- Si: $\theta \in$ IVC

  $\mathbf{Cot\theta < 0}$

Ejemplo:

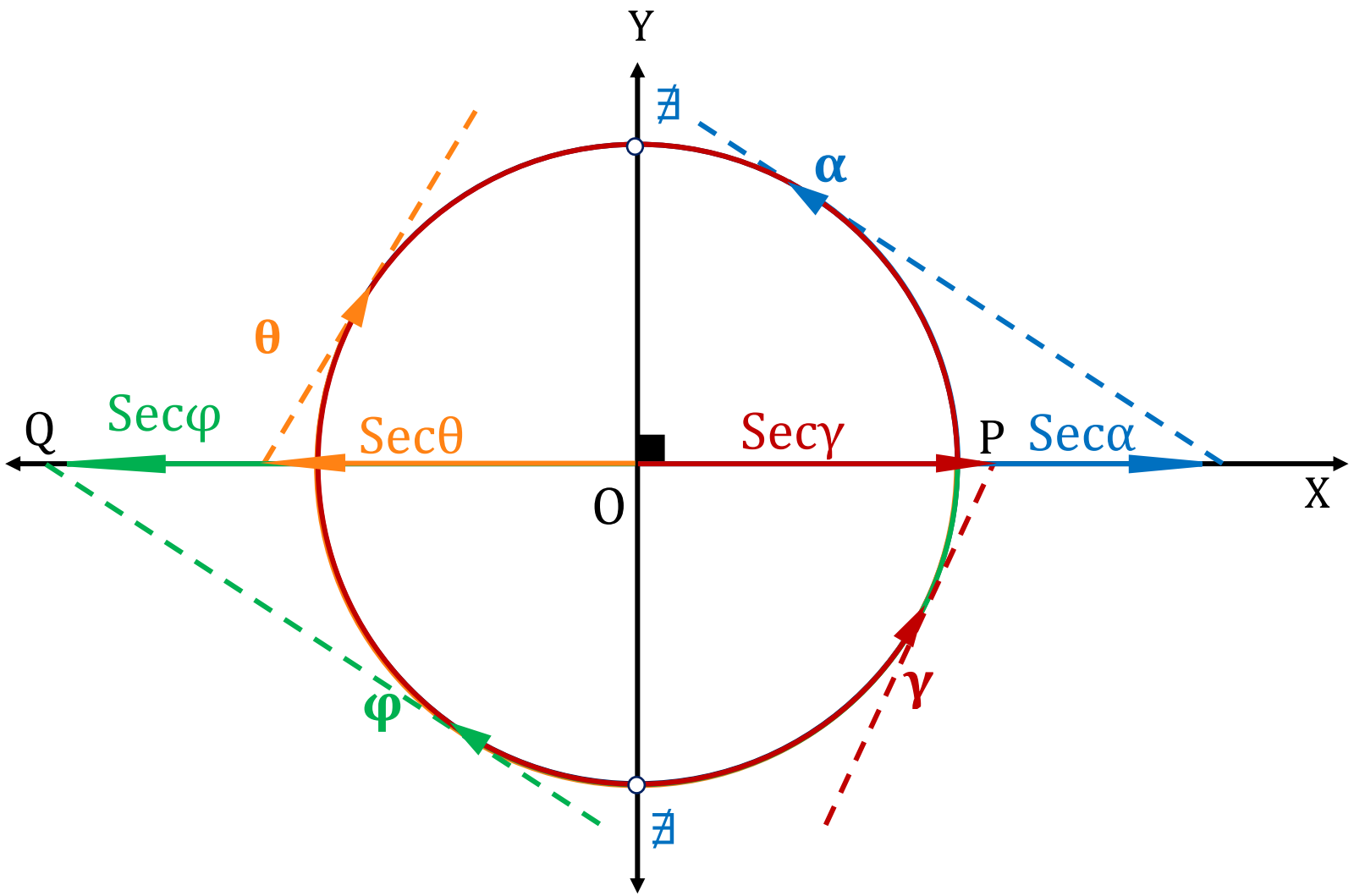
Determinar la extensión de la Cotθ si $\theta \in \left[\frac{4\pi}{3}; \frac{11\pi}{6}\right]$

Resolución:



$$\therefore \mathbf{Cot\theta} \in \left[-\sqrt{3}; \frac{\sqrt{3}}{3}\right]$$

4.5) Representación de la línea Secante:



✓ $\theta < \varphi < \alpha < \gamma$

✓ $\text{Sec}\varphi < \text{Sec}\theta < \text{Sec}\gamma < \text{Sec}\alpha$

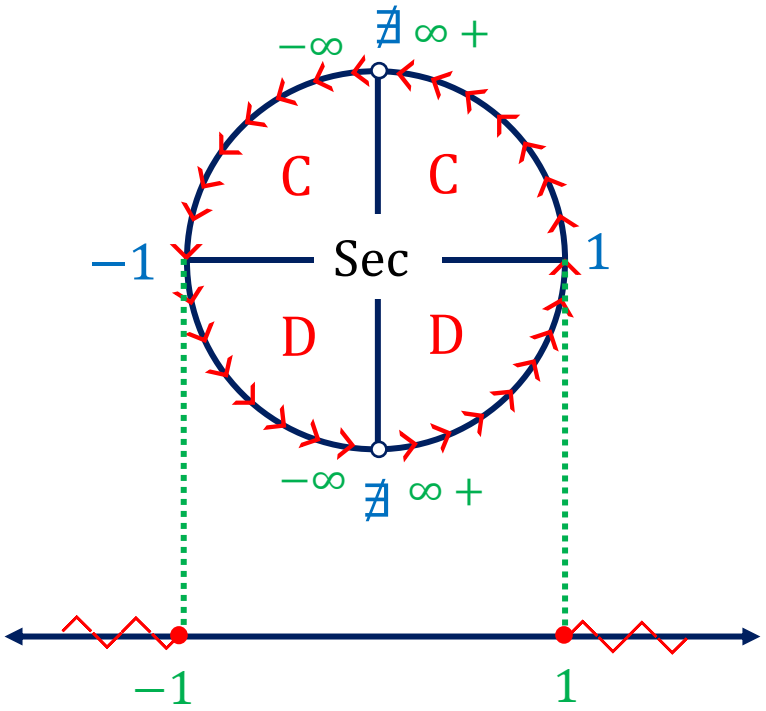
i. Valor $\begin{cases} (+): \text{Sec}\alpha \\ (-): \text{Sec}\theta \end{cases}$

ii. Longitud: $(+)$, $|\text{Secante}|$

$\overline{OP} = |\text{Sec}\gamma| = \text{Sec}\gamma$

$\overline{OQ} = |\text{Sec}\varphi| = -\text{Sec}\varphi$

4.5.1)Variación analítica :



i. Extensión:

$$\forall\, \theta \in \mathbb{R} - \left\{(2n+1)\frac{\pi}{2}; n \in \mathbb{Z}\right\}$$

$$-\infty < \text{Sec}\theta \leq -1 \;\vee\; 1 \leq \text{Sec}\theta < \infty +$$

- Si: $\theta \in$ IC

  $\mathbf{1 < Sec\theta}$

- Si: $\theta \in$ IIC

  $\mathbf{Sec\theta < -1}$

- Si: $\theta \in$ IIIC

  $\mathbf{Sec\theta < -1}$

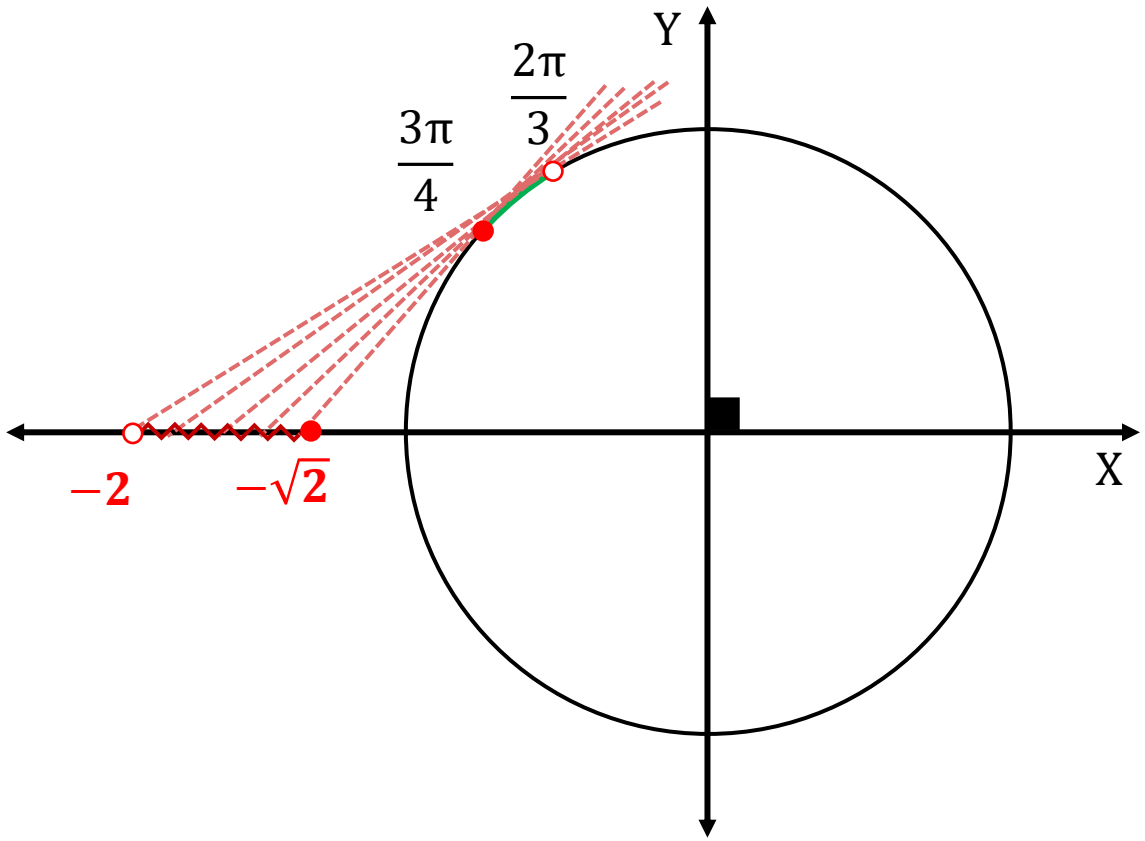
- Si: $\theta \in$ IVC

  $\mathbf{1 < Sec\theta}$

Ejemplo:

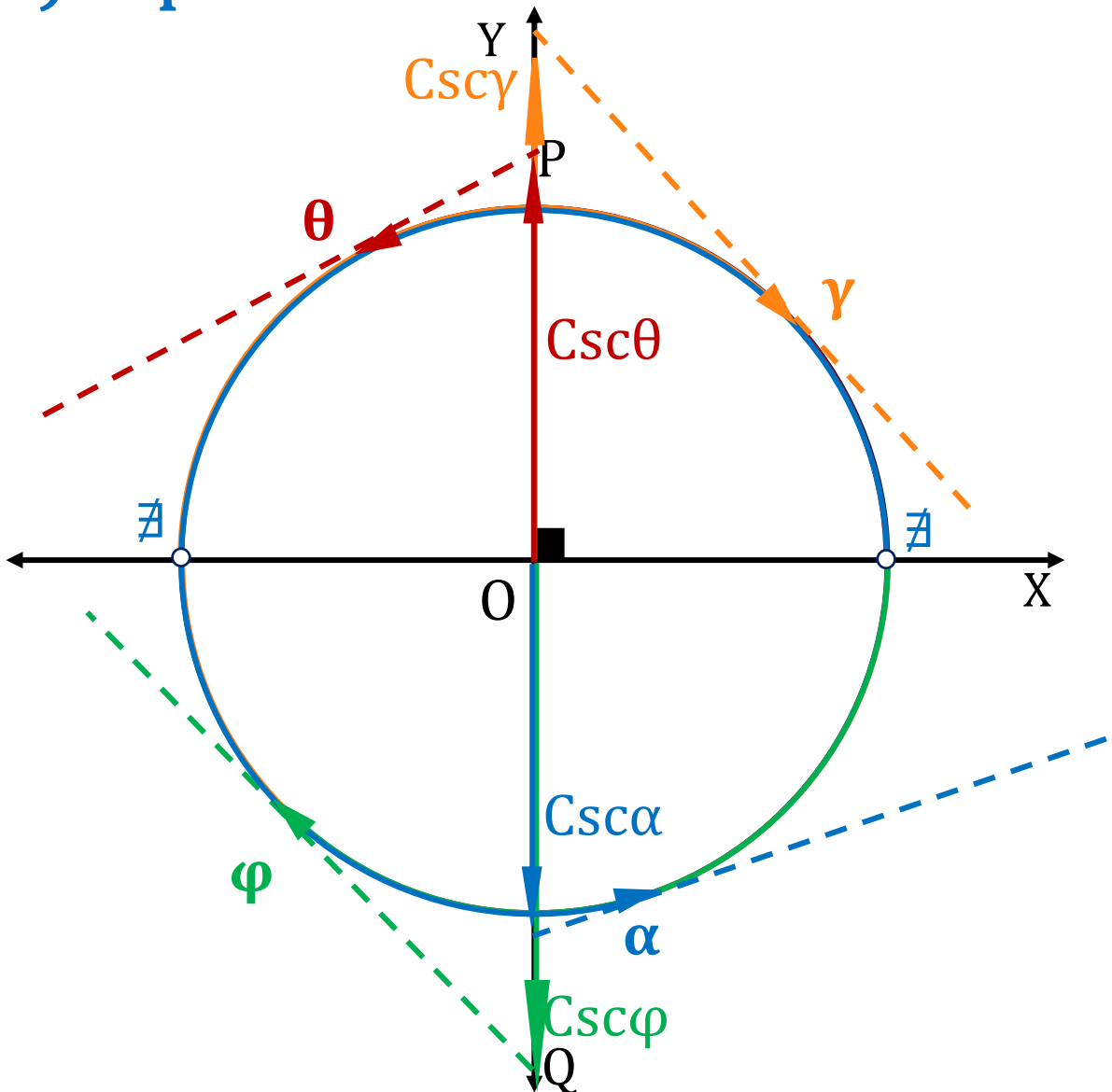
Determinar la extensión de la Secθ si $\theta \in \left]\frac{2\pi}{3};\frac{3\pi}{4}\right]$

Resolución:



$$\therefore \mathbf{Sec\theta} \in \left]-2;-\sqrt{2}\right]$$

4.6) Representación de la línea Cosecante:



✓ $\gamma < \varphi < \theta < \alpha$

✓ $\mathrm{Csc}\varphi < \mathrm{Csc}\alpha < \mathrm{Csc}\theta < \mathrm{Csc}\gamma$

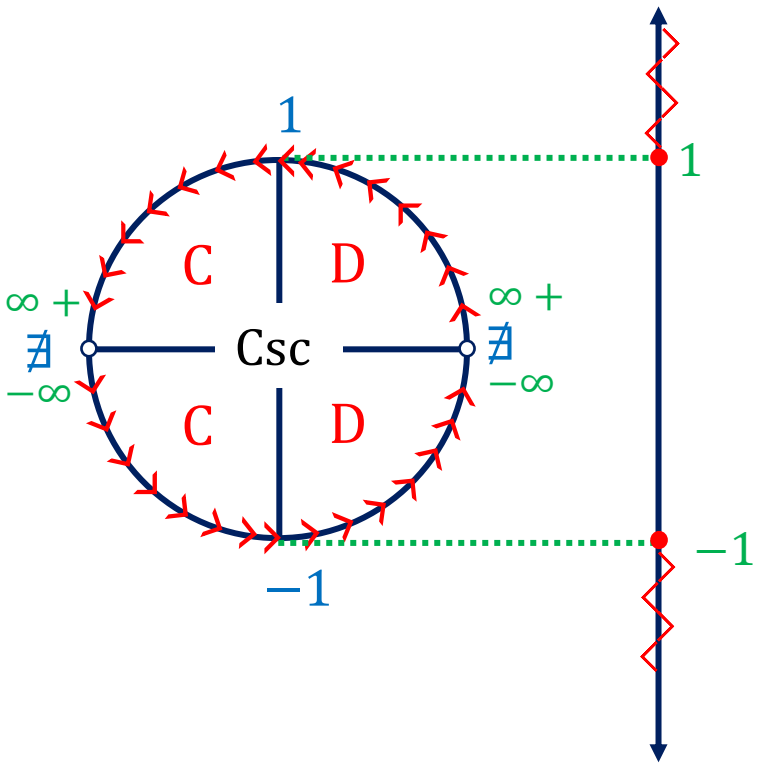
i. Valor $\begin{cases} (+): \mathrm{Csc}\gamma \\ (-): \mathrm{Csc}\alpha \end{cases}$

ii. Longitud: $(+)$, $|\text{Cosecante}|$

$\overline{OP} = |\mathrm{Csc}\theta| = \mathrm{Csc}\theta$

$\overline{OQ} = |\mathrm{Csc}\varphi| = -\mathrm{Csc}\varphi$

4.6.1)Variación analítica :



i. Extensión:

$\forall\, \theta \in \mathbb{R} - \{n\pi; n \in \mathbb{Z}\}$

$-\infty < \text{Csc}\theta \leq -1 \;\; \vee \; 1 \leq \text{Csc}\theta < \infty +$

- Si: $\theta \in$ IC

  $\mathbf{1 < Csc\theta}$

- Si: $\theta \in$ IIC

  $\mathbf{1 < Csc\theta}$

- Si: $\theta \in$ IIIC

  $\mathbf{Csc\theta < -1}$

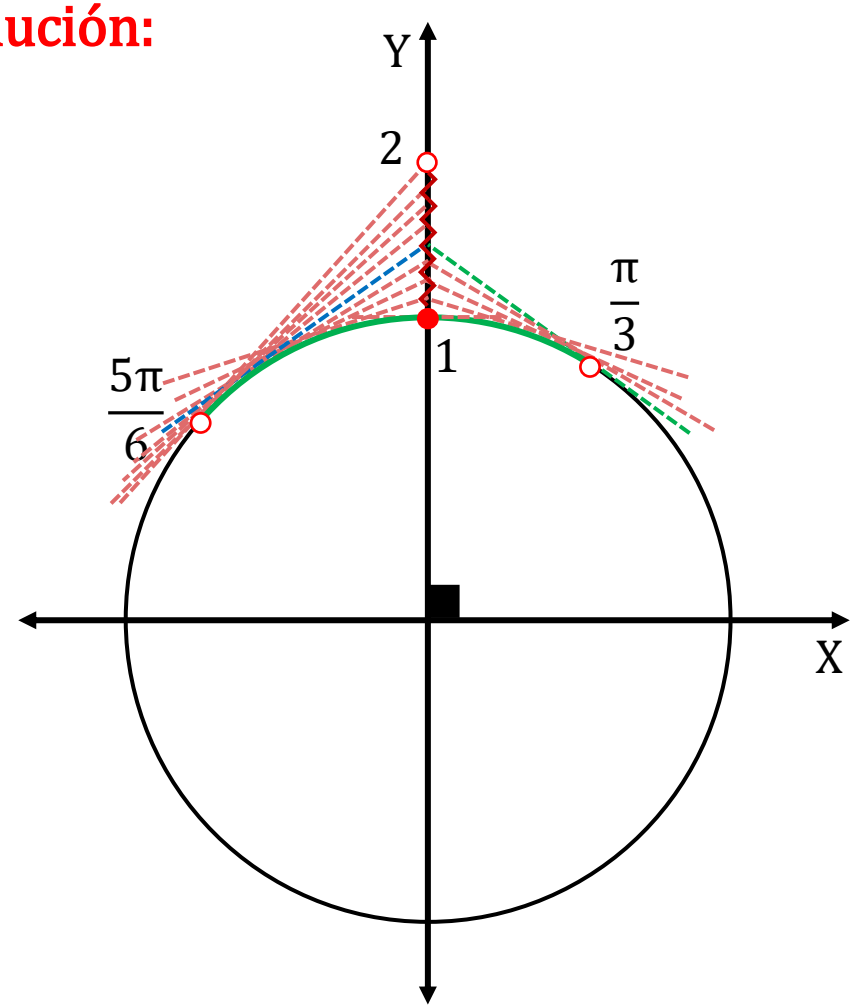
- Si: $\theta \in$ IVC

  $\mathbf{Csc\theta < -1}$

Ejemplo:

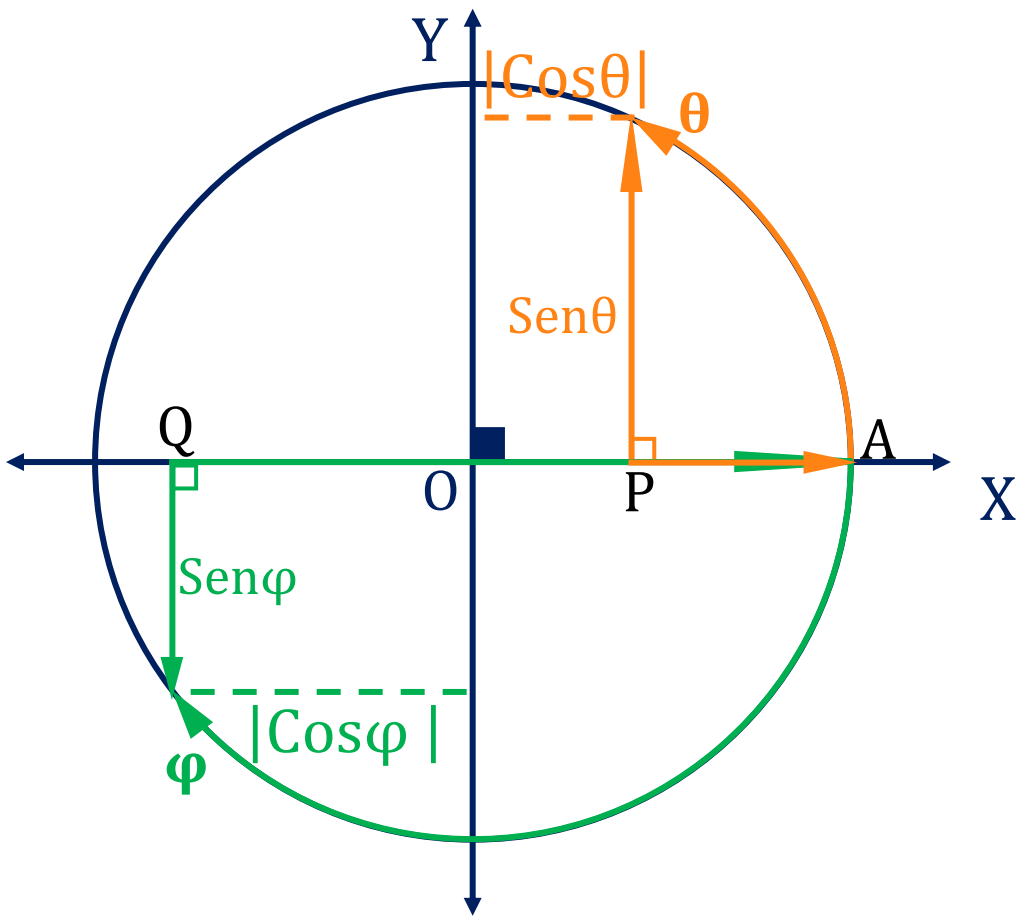
Determinar la extensión de la Cscθ si $\theta \in \left]\frac{\pi}{3};\frac{5\pi}{6}\right[$

Resolución:



$\therefore$ **Cscθ** $\in$ **[1; 2[**

4.7) Representación de la línea Verso o Seno Verso:



$$\text{Vers}\theta = \overrightarrow{PA}$$

$$\text{Vers}\theta = \overline{OA} - \overline{OP}$$

$$\text{Vers}\theta = 1 - |\text{Cos}\theta|$$

$$\text{Vers}\theta = 1 - \text{Cos}\theta$$

$$\text{Vers}\varphi = \overrightarrow{QA}$$

$$\text{Vers}\varphi = \overline{QO} + \overline{OA}$$

$$\text{Vers}\varphi = |\text{Cos}\varphi| + 1$$

$$\text{Vers}\varphi = -\text{Cos}\varphi + 1$$

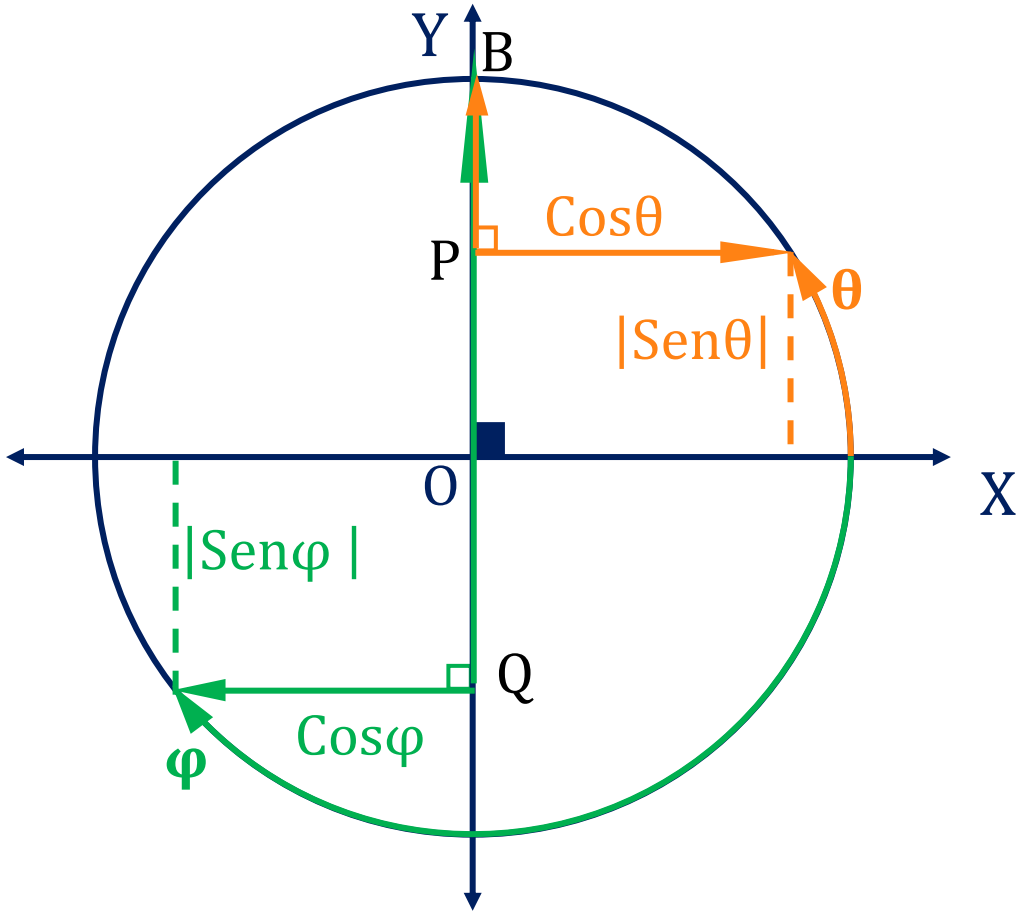
- En general:

$$\text{Vers}x = 1 - \text{Cos}x$$

$$\forall\, x \in \mathbb{R}:$$

$$0 \leq \text{Vers}x \leq 2$$

4.8) Representación de la línea Coverso o Coseno Verso:



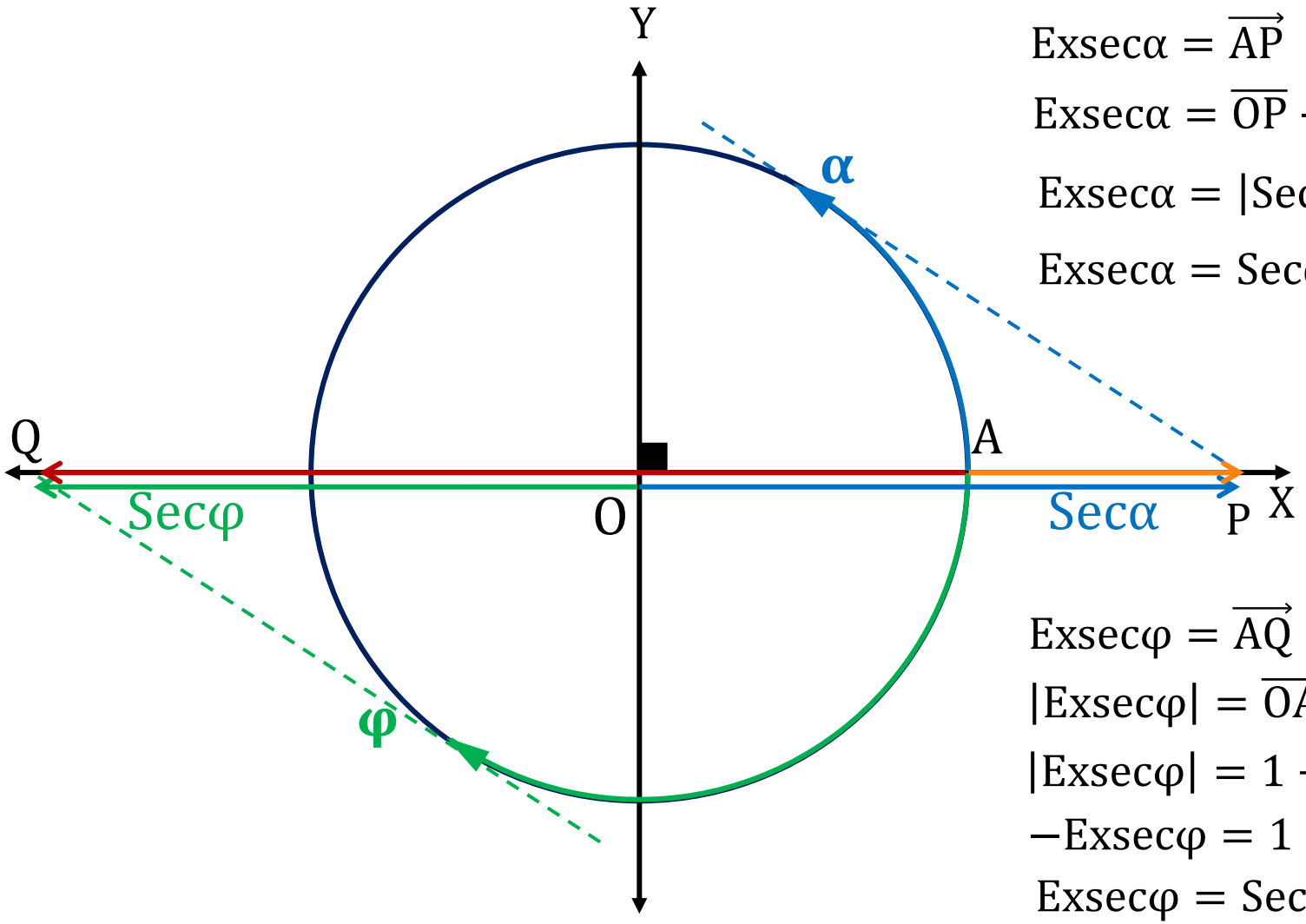
$$\mathrm{Cov}\theta = \overrightarrow{PB}$$
$$\mathrm{Cov}\theta = \overline{OB} - \overline{OP}$$
$$\mathrm{Cov}\theta = 1 - |\mathrm{Sen}\theta|$$
$$\mathrm{Cov}\theta = 1 - \mathrm{Sen}\theta$$

$$\mathrm{Cov}\varphi = \overrightarrow{QB}$$
$$\mathrm{Cov}\varphi = \overline{QO} + \overline{OB}$$
$$\mathrm{Cov}\varphi = |\mathrm{Sen}\varphi| + 1$$
$$\mathrm{Cov}\varphi = -\mathrm{Sen}\varphi + 1$$

- En general:

$$\mathrm{Cov}x = 1 - \mathrm{Sen}x$$
$$\forall\, x \in \mathbb{R}:$$
$$0 \leq \mathrm{Cov}x \leq 2$$

4.9) Representación de la línea Exsecante o External:



$$\text{Exsec}\alpha = \overrightarrow{AP}$$

$$\text{Exsec}\alpha = \overline{OP} - \overline{OA}$$

$$\text{Exsec}\alpha = |\text{Sec}\alpha| - 1$$

$$\text{Exsec}\alpha = \text{Sec}\alpha - 1$$

$$\text{Exsec}\varphi = \overrightarrow{AQ}$$

$$|\text{Exsec}\varphi| = \overline{OA} + \overline{OQ}$$

$$|\text{Exsec}\varphi| = 1 + |\text{Sec}\varphi|$$

$$-\text{Exsec}\varphi = 1 - \text{Sec}\varphi$$

$$\text{Exsec}\varphi = \text{Sec}\varphi - 1$$

- En general:

$$\text{Exsec}x = \text{Sec}x - 1$$

$$\forall\, x \in \mathbb{R} - \left\{(2n+1)\frac{\pi}{2}; n \in \mathbb{Z}\right\}$$

$$\text{Exsec}x \leq -2 \;\vee\; 0 \leq \text{Exsec}x$$

MOMENTO DE PRACTICAR

PROBLEMAS Y RESOLUCIÓN

1. Sabiendo que: $\frac{5}{4} \leq \frac{2+3\text{Tan}\theta}{3\text{Tan}\theta+1} \leq \frac{3}{2}$ Hallar $\text{Csc}\theta_1$, si θ_1 es el mínimo valor de θ positivo, además $\theta \in$ IIIC

Resolución

$$\frac{5}{4} \leq \frac{3\text{Tan}\theta+2}{3\text{Tan}\theta+1} \leq \frac{3}{2}$$

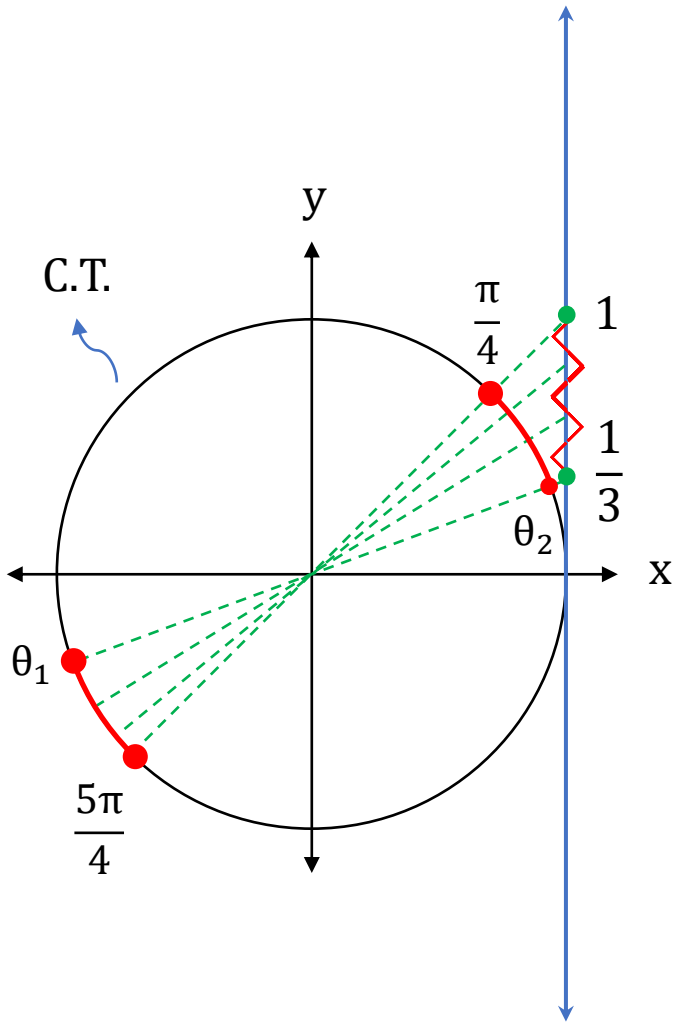
$$\frac{5}{4} \leq 1 + \frac{1}{3\text{Tan}\theta+1} \leq \frac{3}{2}$$

$$\frac{1}{4} \leq \frac{1}{3\text{Tan}\theta+1} \leq \frac{1}{2}$$

$$2 \leq 3\text{Tan}\theta+1 \leq 4$$

$$1 \leq 3\text{Tan}\theta \leq 3$$

$$\frac{1}{3} \leq \text{Tan}\theta \leq 1$$



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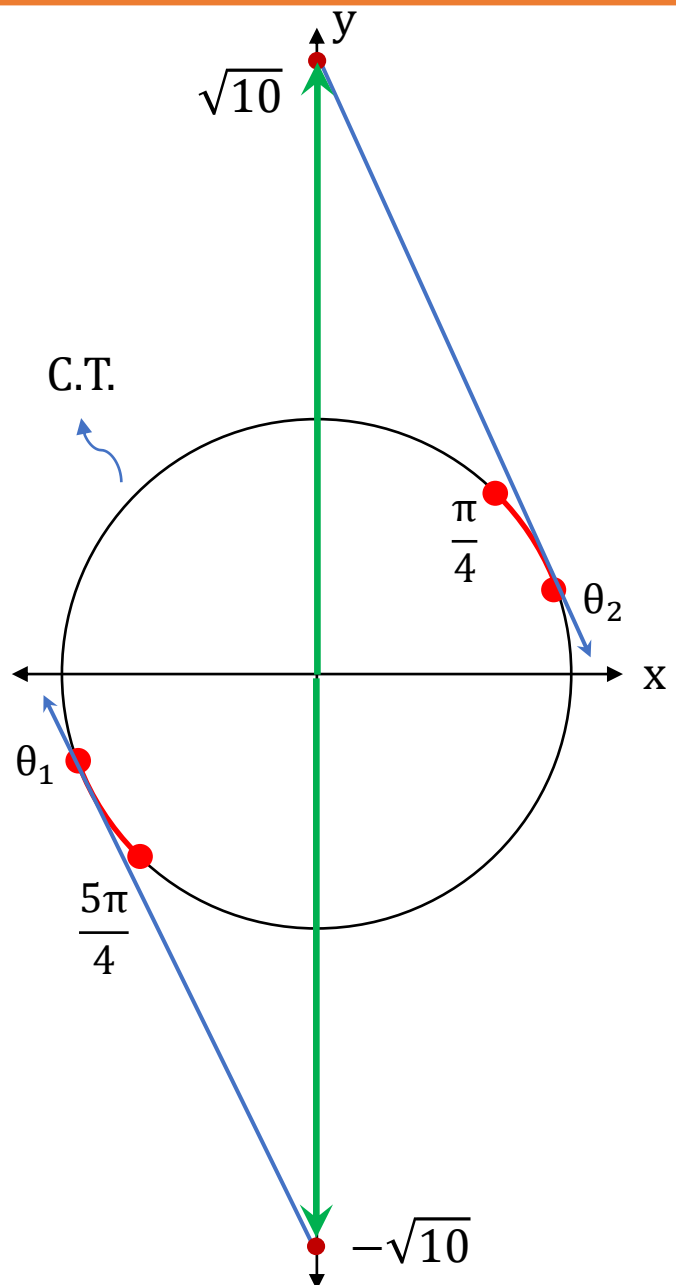
Hallar $\text{Csc}\theta_1$

Resolución



$$\therefore \text{Csc}\theta_1 = -\sqrt{10}$$

CLAVE: C

2. Determinar entre qué límites debe estar comprendido "k" para que sean posibles las siguientes igualdades en forma simultánea :

$$\text{Csc}x = \frac{5k-1}{2}; x \in \text{IIIC} \qquad \text{Co}ty = \frac{k^2-9}{7}; y \in \text{IVC}$$

Resolución

$x \in \text{IIIC} \rightarrow \text{Csc}x < -1$

$$\frac{5k-1}{2} < -1$$

$$5k-1 < -2$$

$$5k < -1$$

$$k < -\frac{1}{5}$$

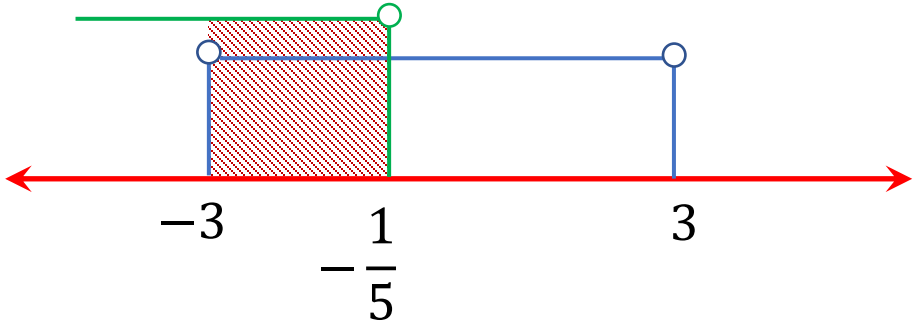
$y \in \text{IVC} \rightarrow \text{Co}ty < 0$

$$\frac{k^2-9}{7} < 0$$

$$k^2 - 9 < 0$$

$$k^2 < 9$$

$$-3 < k < 3$$

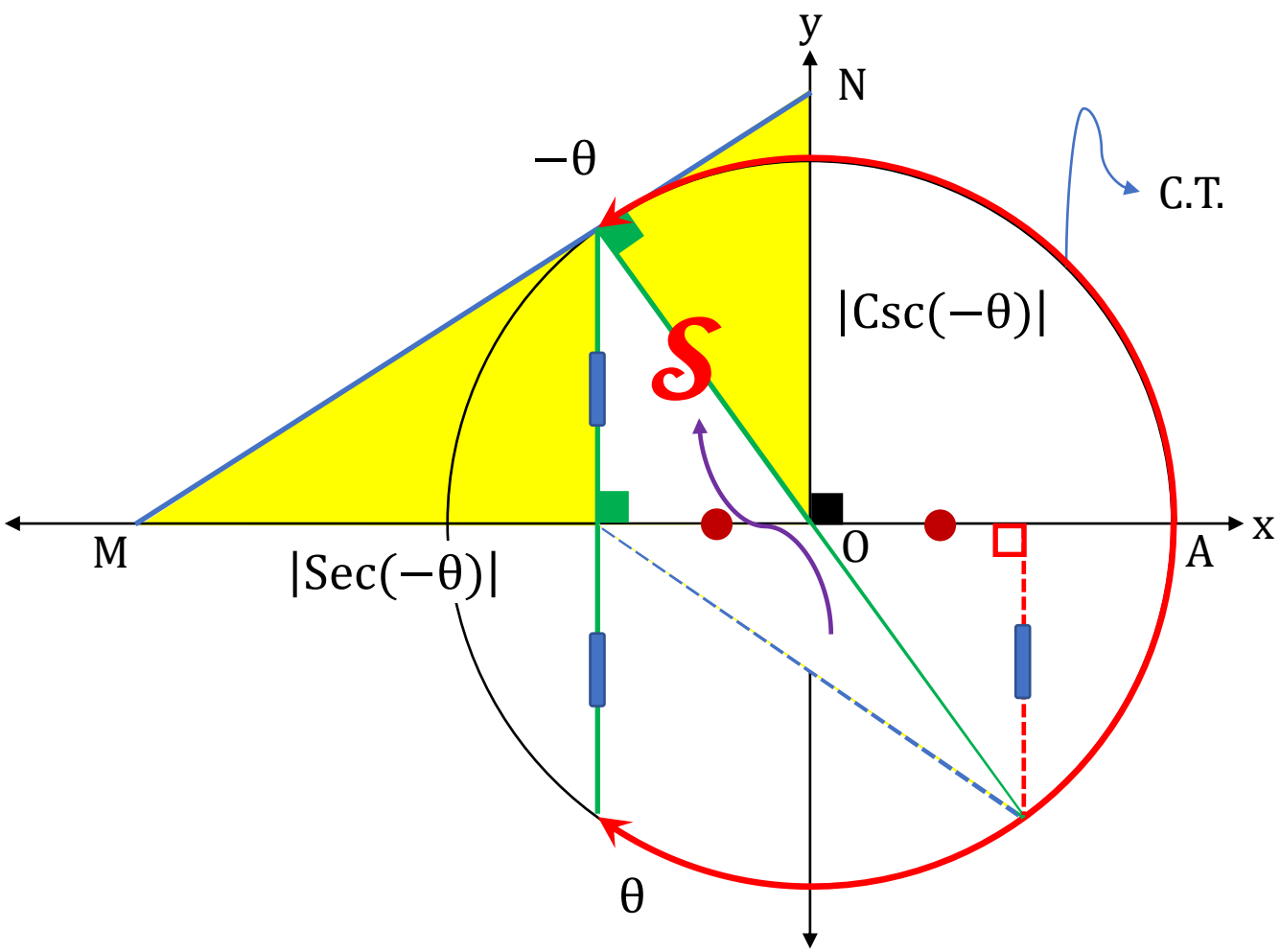


$$\therefore \mathbf{k} \in \left\langle -3; -\frac{1}{5} \right\rangle$$

CLAVE: C

3. En la C.T mostrada calcular el mínimo valor de la región sombreada

Resolución:



$$S = \frac{1}{2}|\mathrm{Sec}\theta||-\mathrm{Csc}\theta|$$

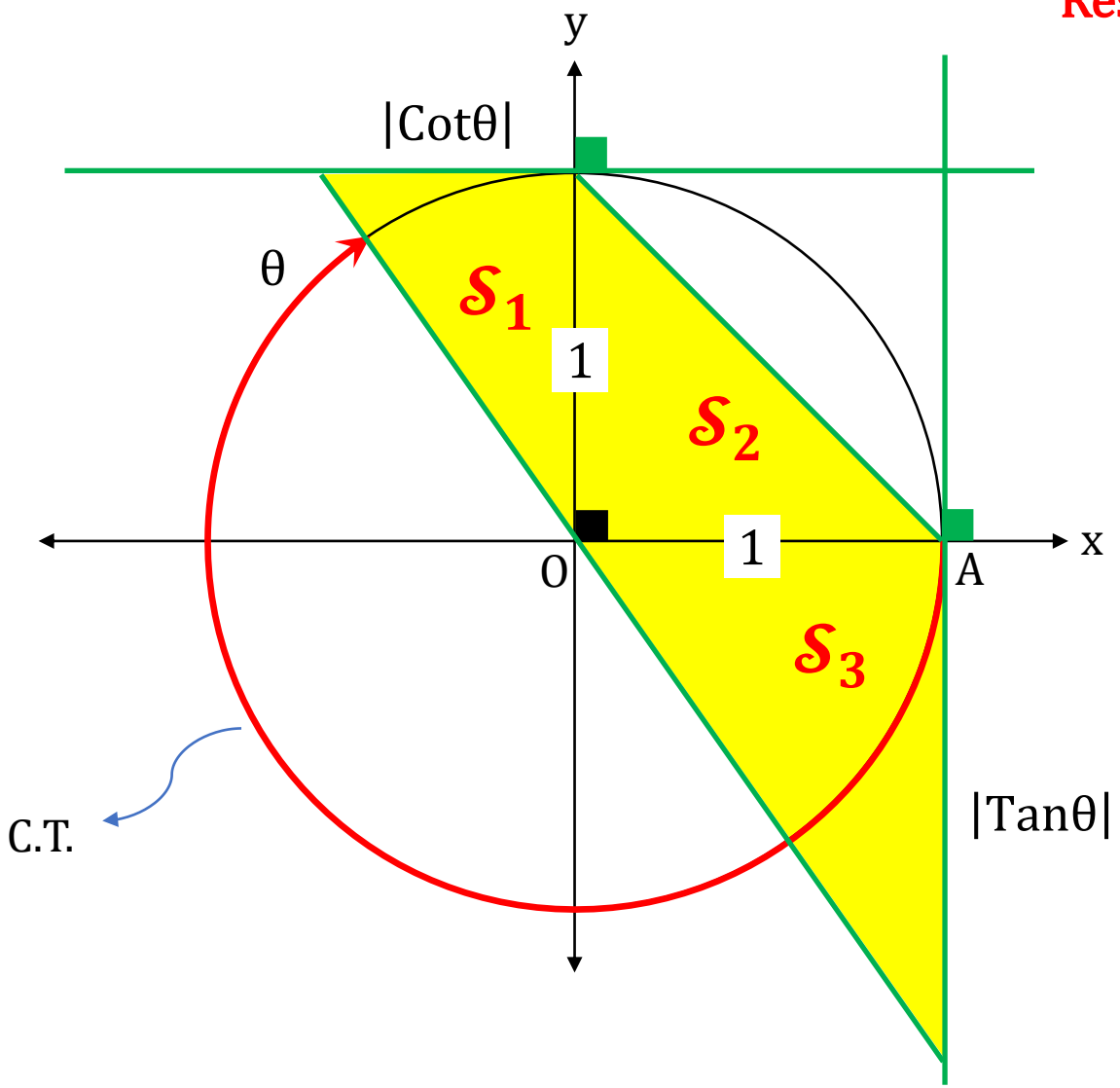
$$S = \frac{1}{2}(-\mathrm{Sec}\theta)(-\mathrm{Csc}\theta)$$

$$S_{mín} = \frac{1}{2}(\overbrace{\mathrm{Tan}\theta + \mathrm{Cot}\theta}^{2})$$

$$\therefore \mathbf{S_{mín} = 1}$$

CLAVE: D

4. Calcular el área mínima de la región sombreada:



Resolución:

$$\mathcal{S} = \mathcal{S}_1 + \mathcal{S}_2 + \mathcal{S}_3$$

$$\mathcal{S} = \frac{1 \times |\text{Cot}\theta|}{2} + \frac{1.1}{2} + \frac{1 \times |\text{Tan}\theta|}{2}$$

$$\mathcal{S} = \frac{-\text{Cot}\theta}{2} + \frac{1}{2} + \frac{-\text{Tan}\theta}{2}$$

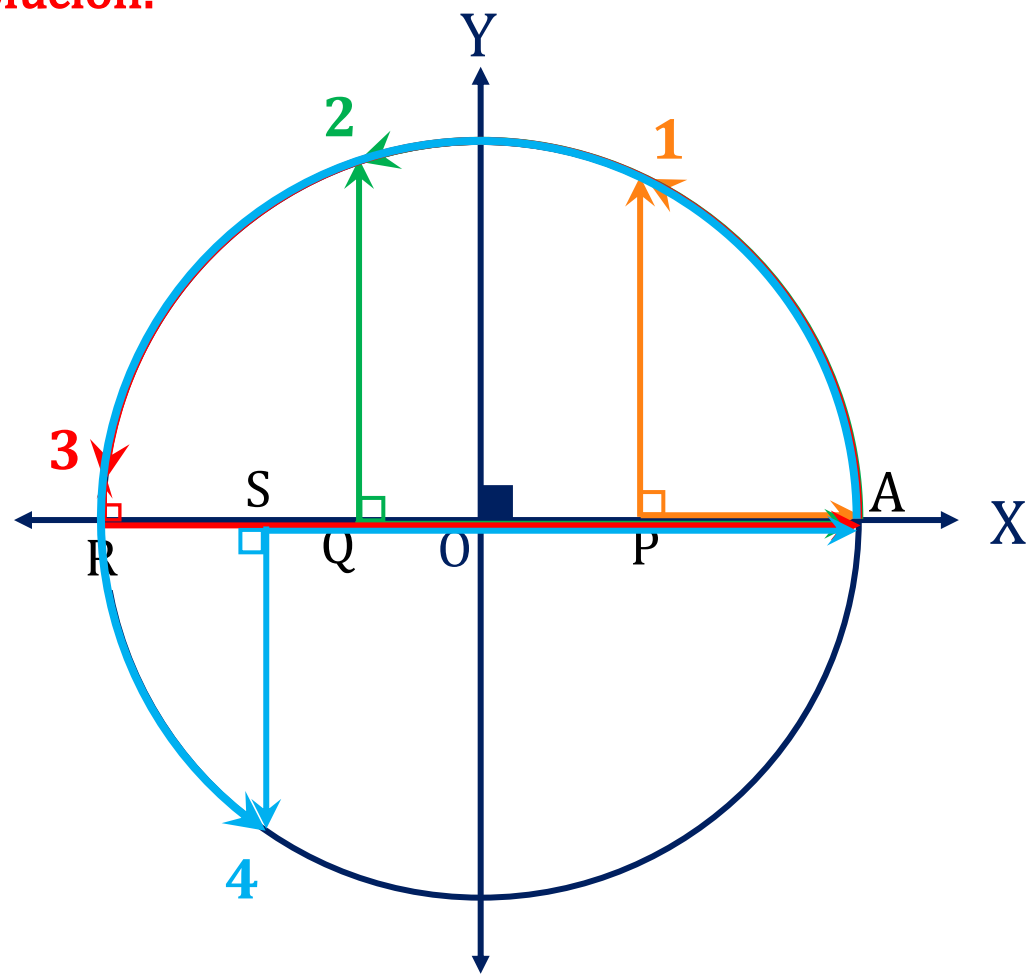
$$\mathcal{S}_{\text{mín}} = \frac{1 - \overbrace{(\text{Tan}\theta + \text{Cot}\theta)}^{-2}}{2}$$

$$\therefore \mathcal{S}_{\text{mín}} = \frac{3}{2}$$

CLAVE: A

5. Ordenar en forma decreciente: Vers(1); Vers(2); Vers(3); Vers(4)

Resolución:



$PA = Vers1$

$QA = Vers2$

$RA = Vers3$

$SA = Vers4$

$\therefore$ **Vers3 > Vers4 > Vers2 > Vers1**

CLAVE: E

6. Sabiendo que: $\sqrt{3}\text{Sen}\theta - \text{Cot}x = 2\text{Cos}^2\varphi$; $\forall\ \theta\ \wedge \varphi \in \mathbb{R}$ calcular la variación de "x" en el intervalo de $< 0; \pi >$

RESOLUCIÓN:

$$\text{Cot}x = \sqrt{3}\text{Sen}\theta - 2\text{Cos}^2\varphi$$

$$-1 \leq \text{Sen}\theta \leq 1$$

$$\left.\begin{array}{c} -\sqrt{3} \leq \sqrt{3}\text{Sen}\theta \leq \sqrt{3} \\ \\ -1 \leq \text{Cos}\varphi \leq 1 \\ 0 \leq \text{Cos}^2\varphi \leq 1 \\ 0 \leq 2\text{Cos}^2\varphi \leq 2 \\ -2 \leq -2\text{Cos}^2\varphi \leq 0 \end{array}\right\} + \qquad -2 - \sqrt{3} \leq \sqrt{3}\text{Sen}\theta - 2\text{Cos}^2\varphi \leq \sqrt{3}$$

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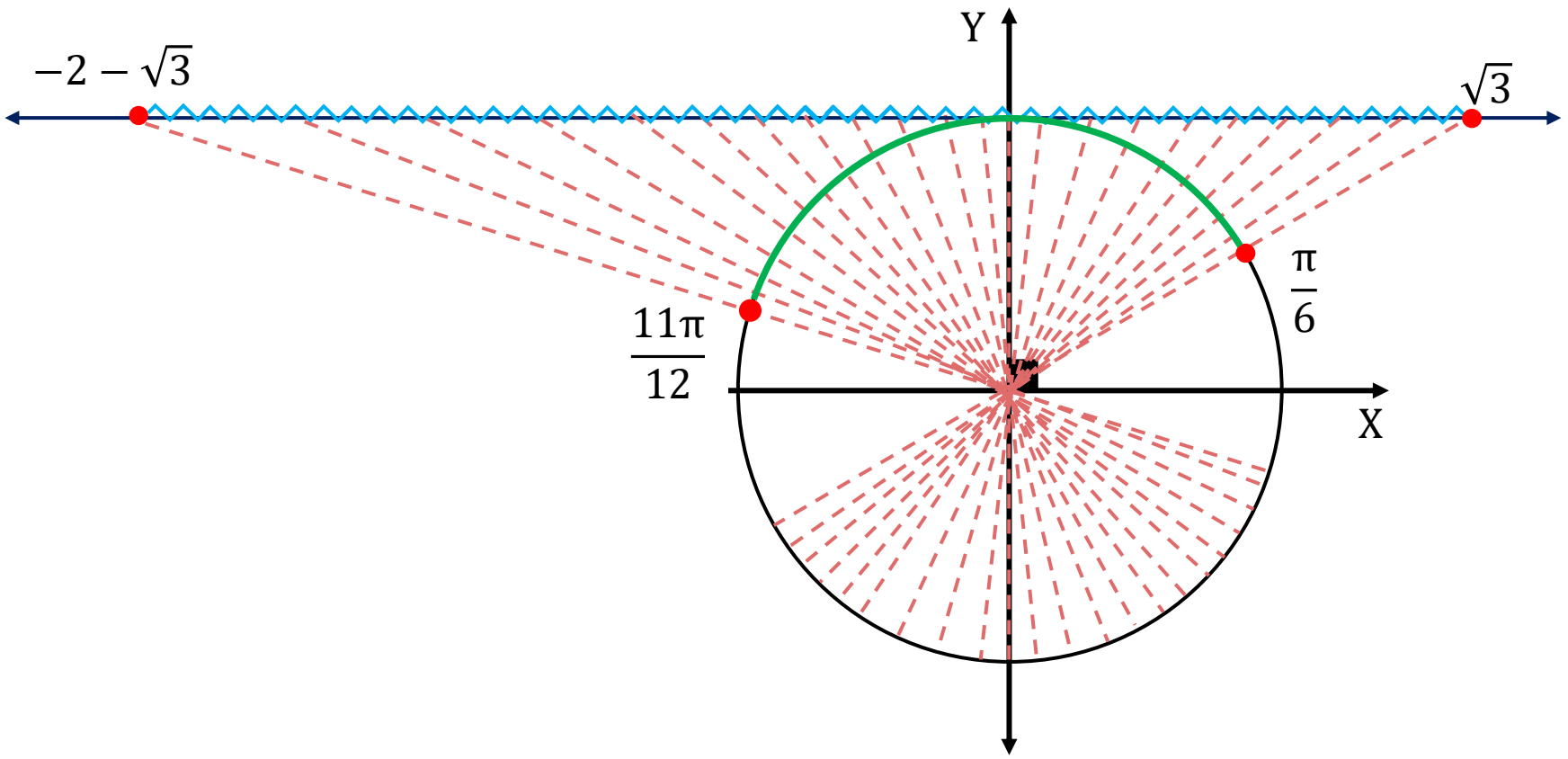
$$-2-\sqrt{3} \leq \text{Cotx} \leq \sqrt{3}$$



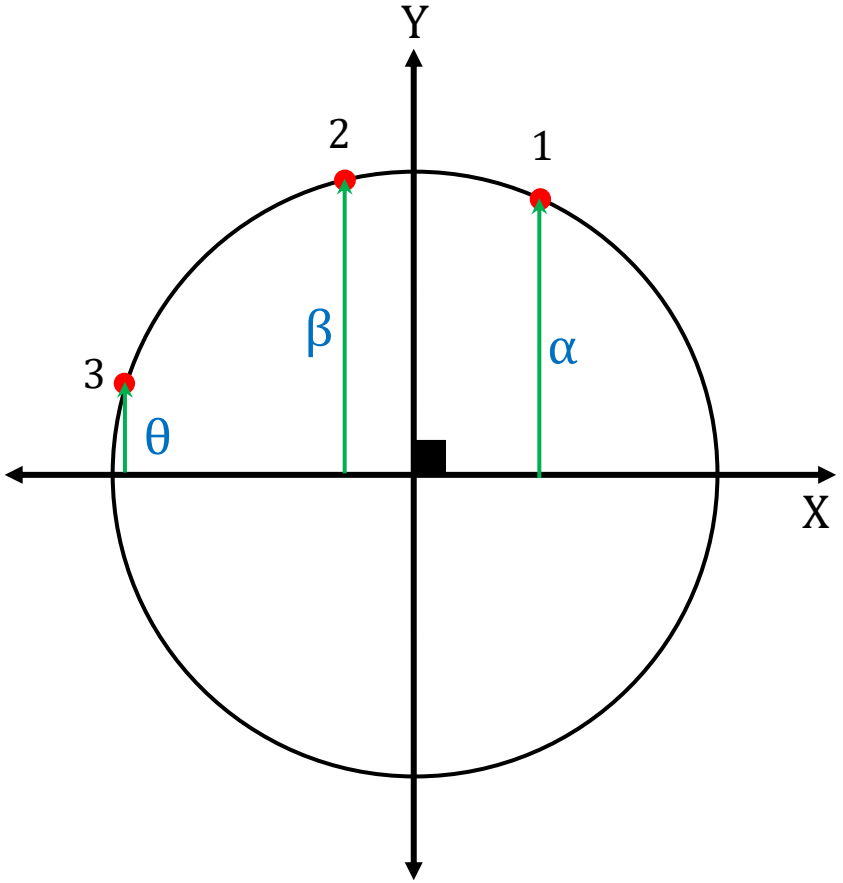
$$\frac{\pi}{6} \leq x \leq \frac{11\pi}{12}$$

$$\therefore \mathbf{x} \in \left[\frac{\boldsymbol{\pi}}{\mathbf{6}}; \frac{\mathbf{11}\boldsymbol{\pi}}{\mathbf{12}}\right]$$

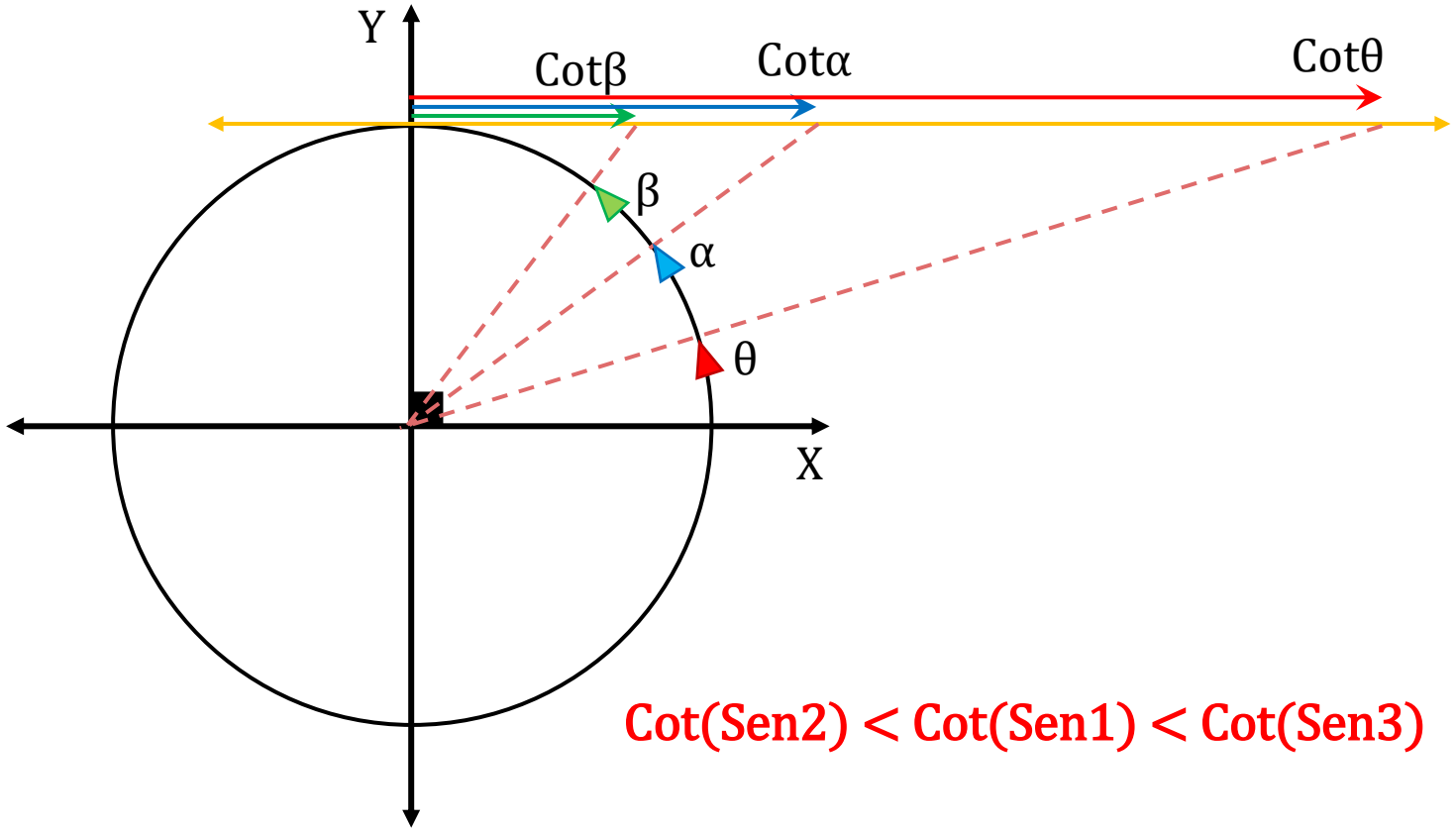
CLAVE: A

7. Ordenar en forma ascendente : Cot(Sen1); Cot(Sen2); Cot(Sen3)

RESOLUCIÓN:



$0 < \theta < \alpha < \beta < 1$



Cot(Sen2) < Cot(Sen1) < Cot(Sen3)

CLAVE: E

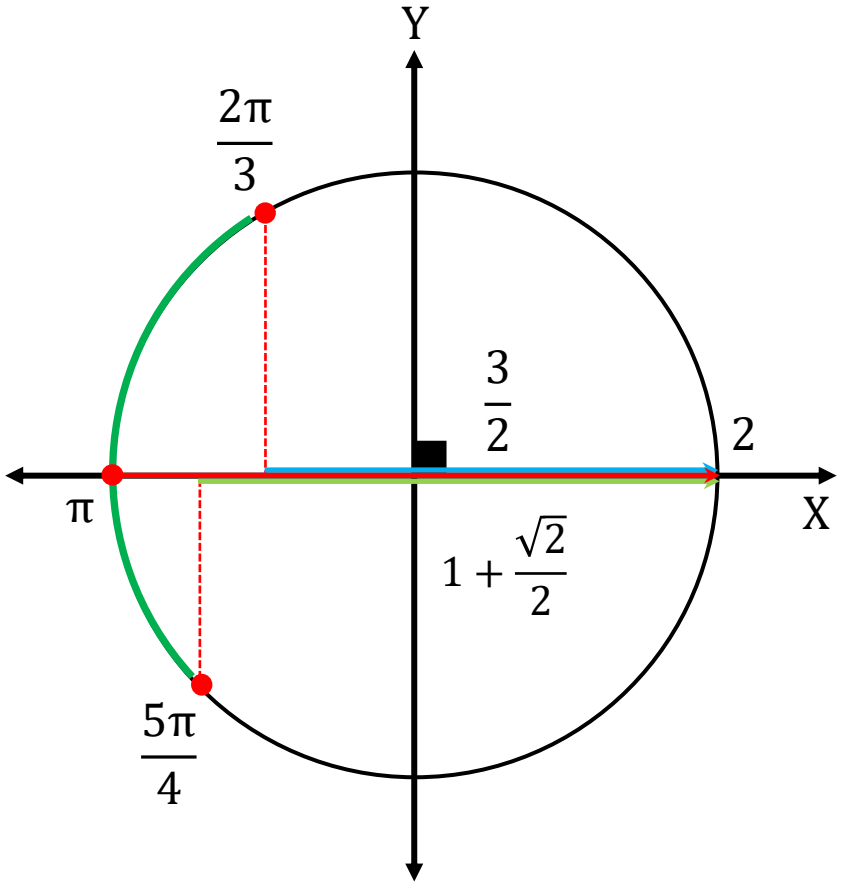
8. Sabiendo que: $\frac{7\pi}{6} \leq x \leq \frac{7\pi}{3}$ calcular la variación de " $\text{Vers}\left(\frac{x}{2}+\frac{\pi}{12}\right)$ "

RESOLUCIÓN:

$$\frac{7\pi}{6} \leq x \leq \frac{7\pi}{3}$$

$$\frac{7\pi}{12} \leq \frac{x}{2} \leq \frac{7\pi}{6}$$

$$\frac{2\pi}{3} \leq \frac{x}{2}+\frac{\pi}{12} \leq \frac{5\pi}{4}$$



$$\frac{3}{2} \leq \text{Vers}\left(\frac{x}{2}+\frac{\pi}{12}\right) \leq 2$$

$$\therefore \left[\frac{3}{2}; 2\right]$$

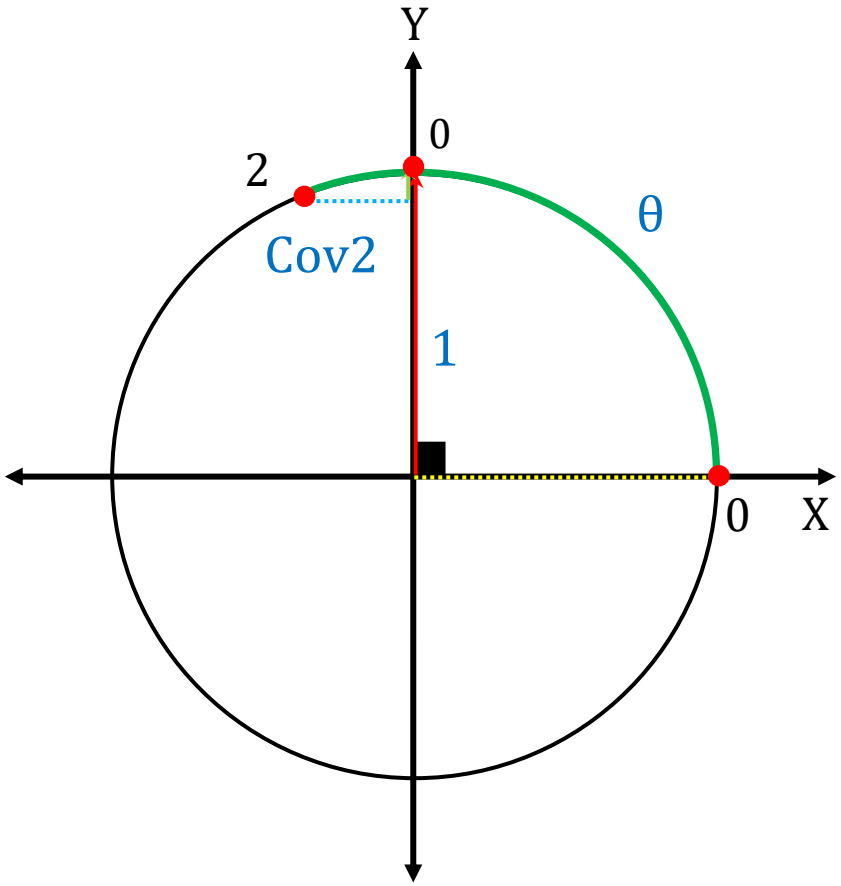
CLAVE: B

9. Hallar la extensión de " Cov(Versx)"

RESOLUCIÓN:

$x \in \mathbb{R}$

$0 \leq \underbrace{\text{Versx}}_{\theta} \leq 2$



$0 \leq \text{Cov(Versx)} \leq 1$

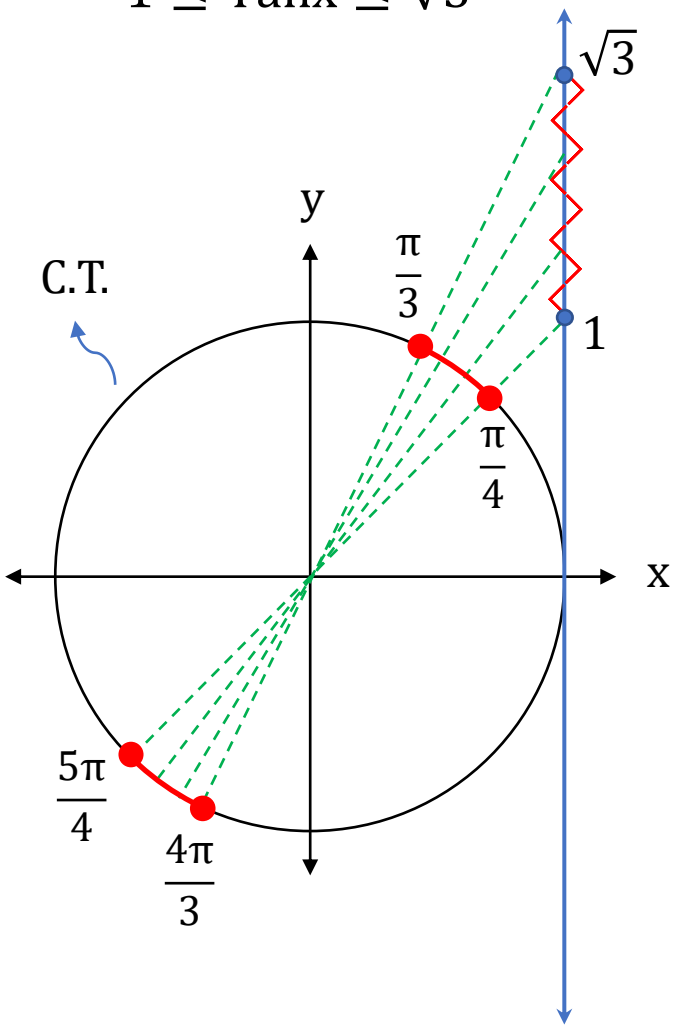
$[\mathbf{0}; \mathbf{1}]$

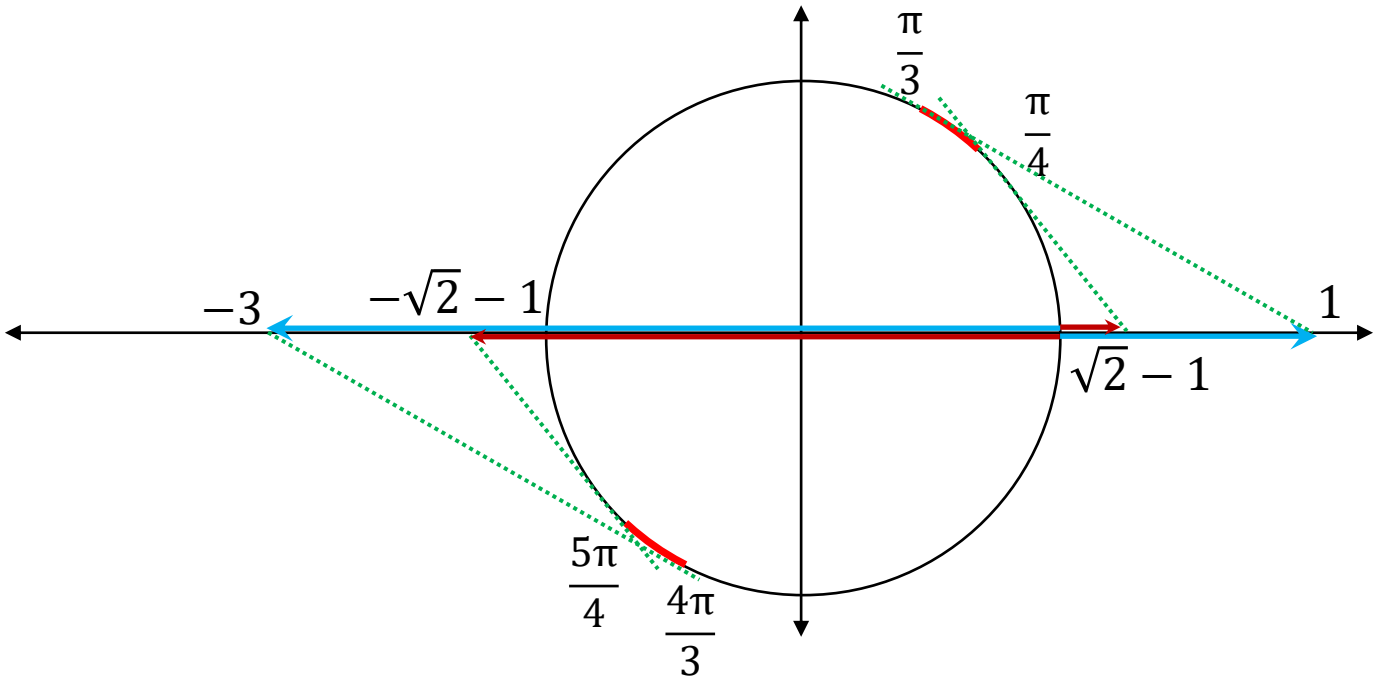
CLAVE: C

10. Hallar la extensión de " ExSecx" Sabiendo que: $1 \leq \text{Tanx} \leq \sqrt{3}$

RESOLUCIÓN:

$$1 \leq \text{Tanx} \leq \sqrt{3}$$





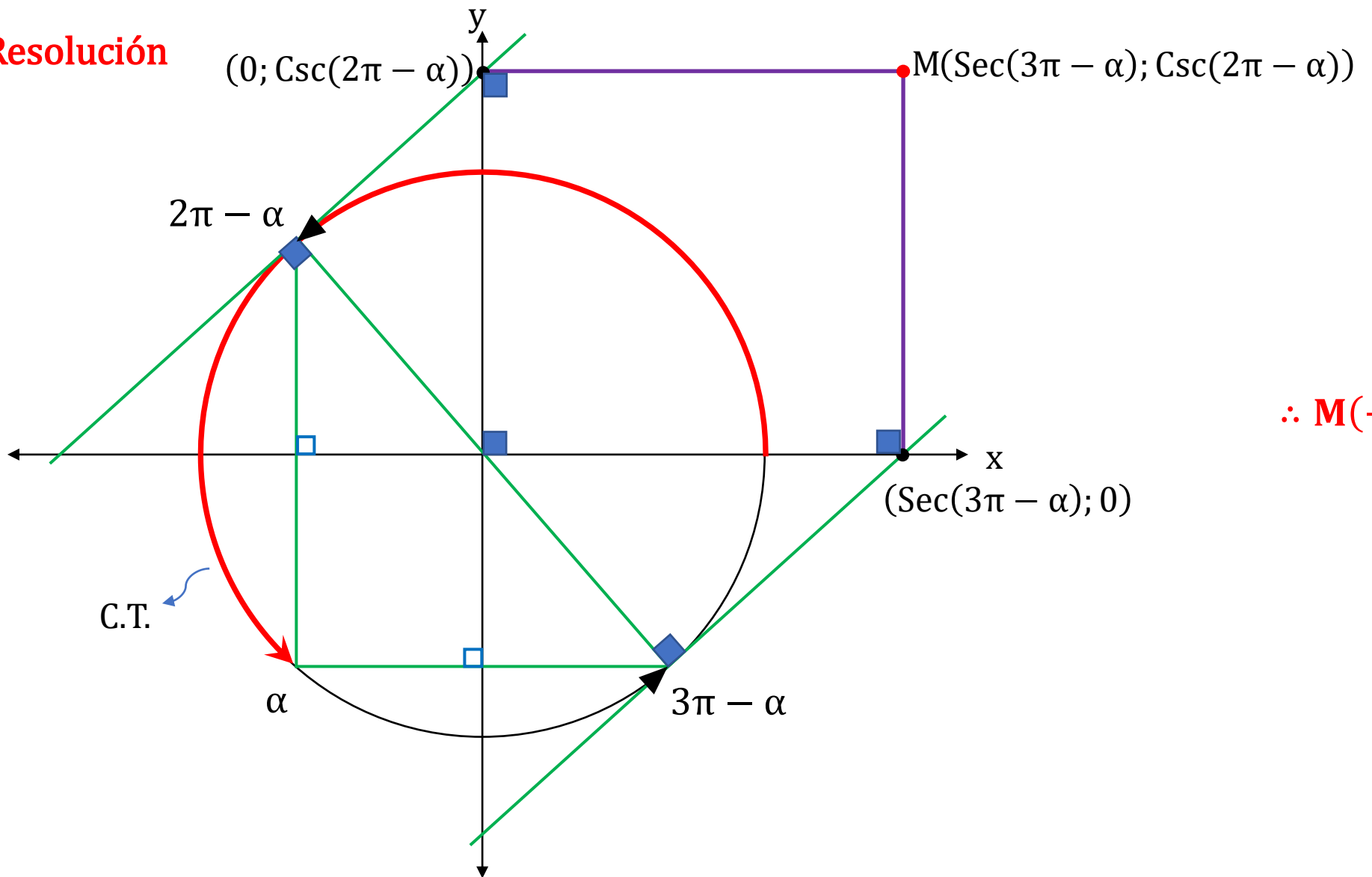
$$-3 \leq \text{ExSecx} \leq -\sqrt{2}-1 \quad \cup \quad \sqrt{2}-1 \leq \text{ExSecx} \leq 1$$

$$\mathbf{[-3; -\sqrt{2}-1] \quad \cup \quad [\sqrt{2}-1; 1]}$$

CLAVE: D

11. Hallar la coordenada del punto M en términos de α en la C.T. mostrada

Resolución



$\therefore$ **$M(-\mathrm{Sec}\alpha; -\mathrm{Csc}\alpha)$**

CLAVE: E

12. Determinar la extensión de : $E = \text{Cot}\left(\frac{\pi}{2}\text{Sen}\frac{x}{2}\right)$, sabiendo que: $x \in \left\langle \frac{5\pi}{3}; \frac{10\pi}{3} \right\rangle$

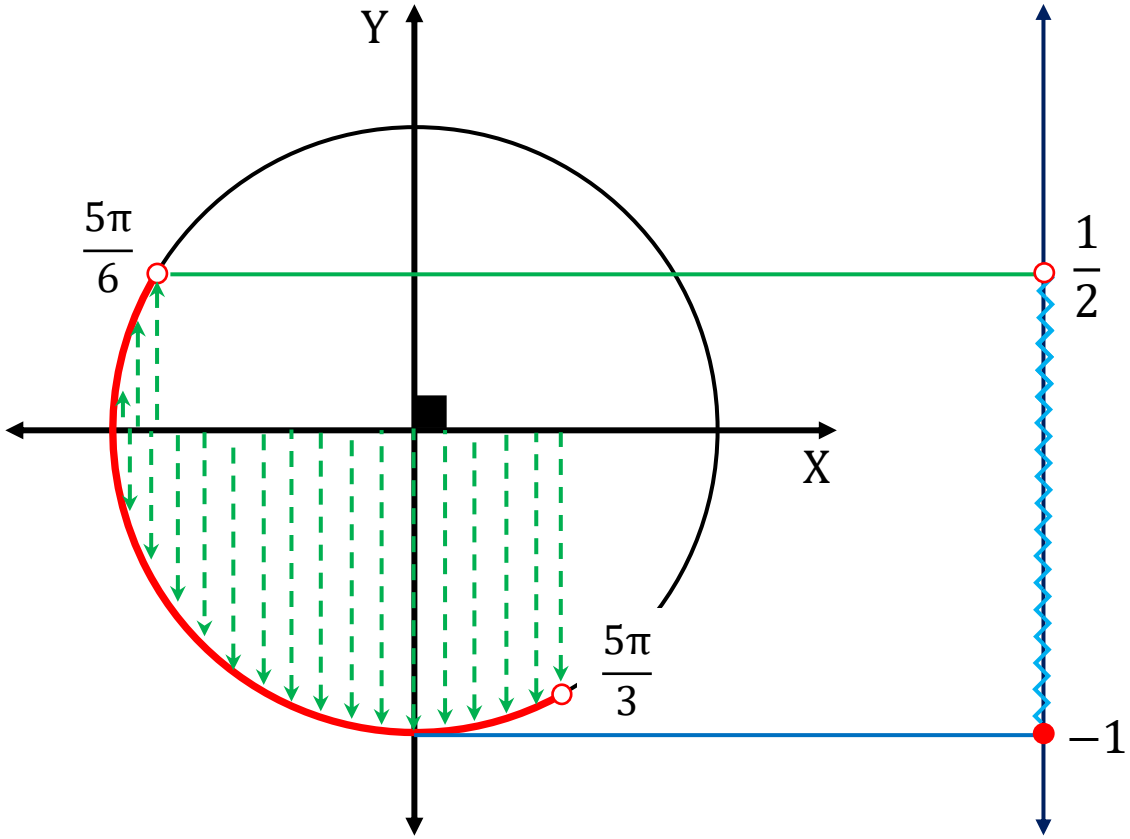
Resolución

$$\frac{5\pi}{3} < x < \frac{10\pi}{3}$$

$$\frac{5\pi}{6} < \frac{x}{2} < \frac{5\pi}{3}$$

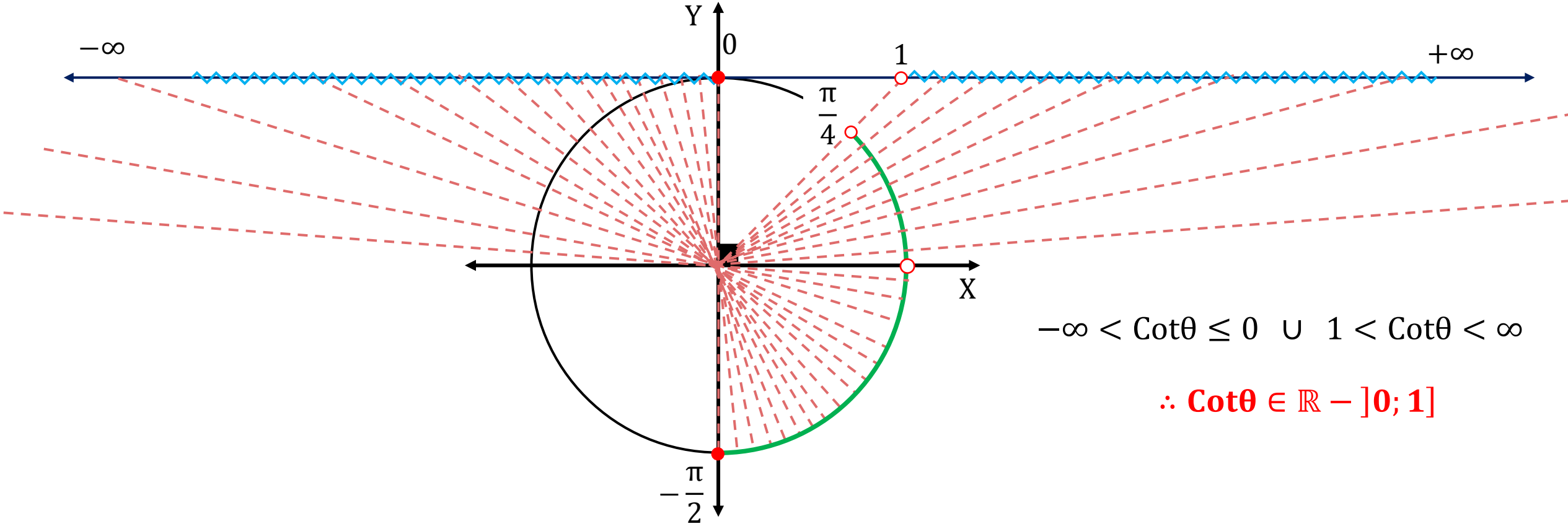
$$-1 \leq \text{Sen}\frac{x}{2} < \frac{1}{2}$$

$$-\frac{\pi}{2} \leq \underbrace{\frac{\pi}{2}\text{Sen}\frac{x}{2}}_{\theta} < \frac{\pi}{4}$$



CIRCUNFERENCIA TRIGONOMÉTRICA II

$$-\frac{\pi}{2} \leq \theta < \frac{\pi}{4} \quad \rightarrow \quad ? < \text{Cot}\theta < \ ?$$



$$-\infty < \text{Cot}\theta \leq 0 \ \cup \ 1 < \text{Cot}\theta < \infty$$

$$\therefore \ \mathbf{Cot\theta} \in \mathbb{R} - \mathbf{]0;1]}$$

CLAVE: B

13. Sabiendo que $x \in \left[\frac{17\pi}{24}; \frac{41\pi}{40}\right]$, hallar el máximo valor que adopta : $\text{Sec}\left(\text{Sen}\left(2x - \frac{\pi}{4}\right)\right)$

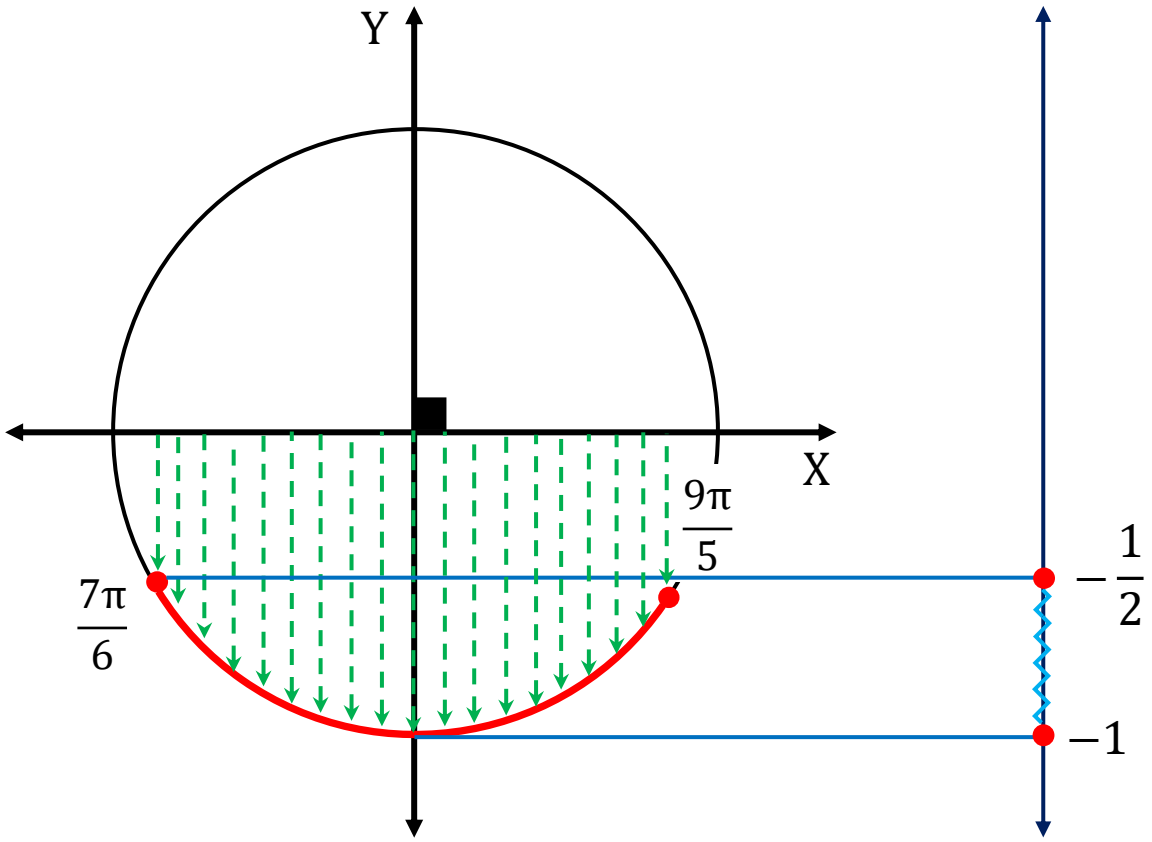
RESOLUCIÓN:

$$\frac{17\pi}{24} \leq x \leq \frac{41\pi}{40}$$

$$\frac{17\pi}{12} \leq 2x \leq \frac{41\pi}{20}$$

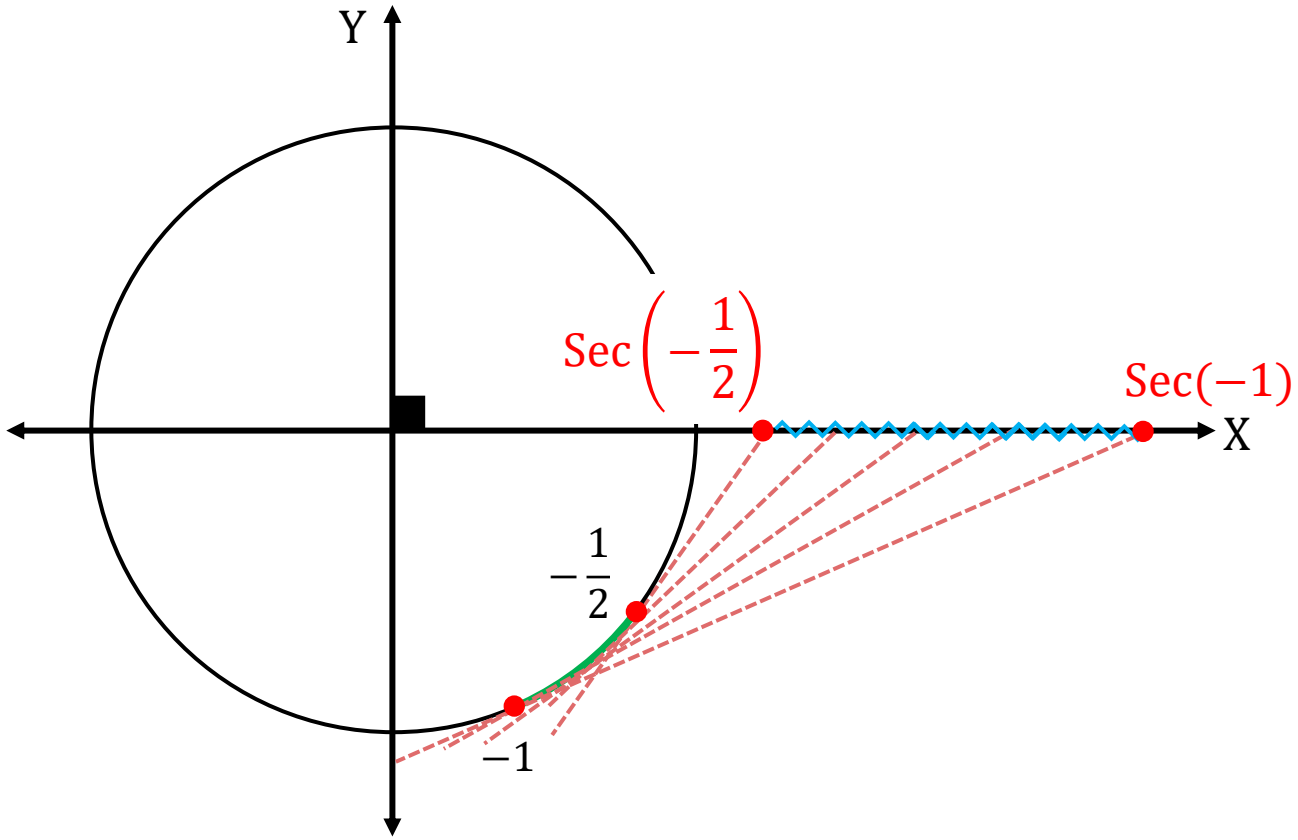
$$\frac{7\pi}{6} \leq 2x - \frac{\pi}{4} \leq \frac{9\pi}{5}$$

$$-1 \leq \underbrace{\text{Sen}\left(2x - \frac{\pi}{4}\right)}_{\theta} \leq -\frac{1}{2}$$



CIRCUNFERENCIA TRIGONOMÉTRICA II

$$-1 \leq \theta \leq -\frac{1}{2} \quad \rightarrow \quad ? < \text{Sec}\theta < \ ?$$



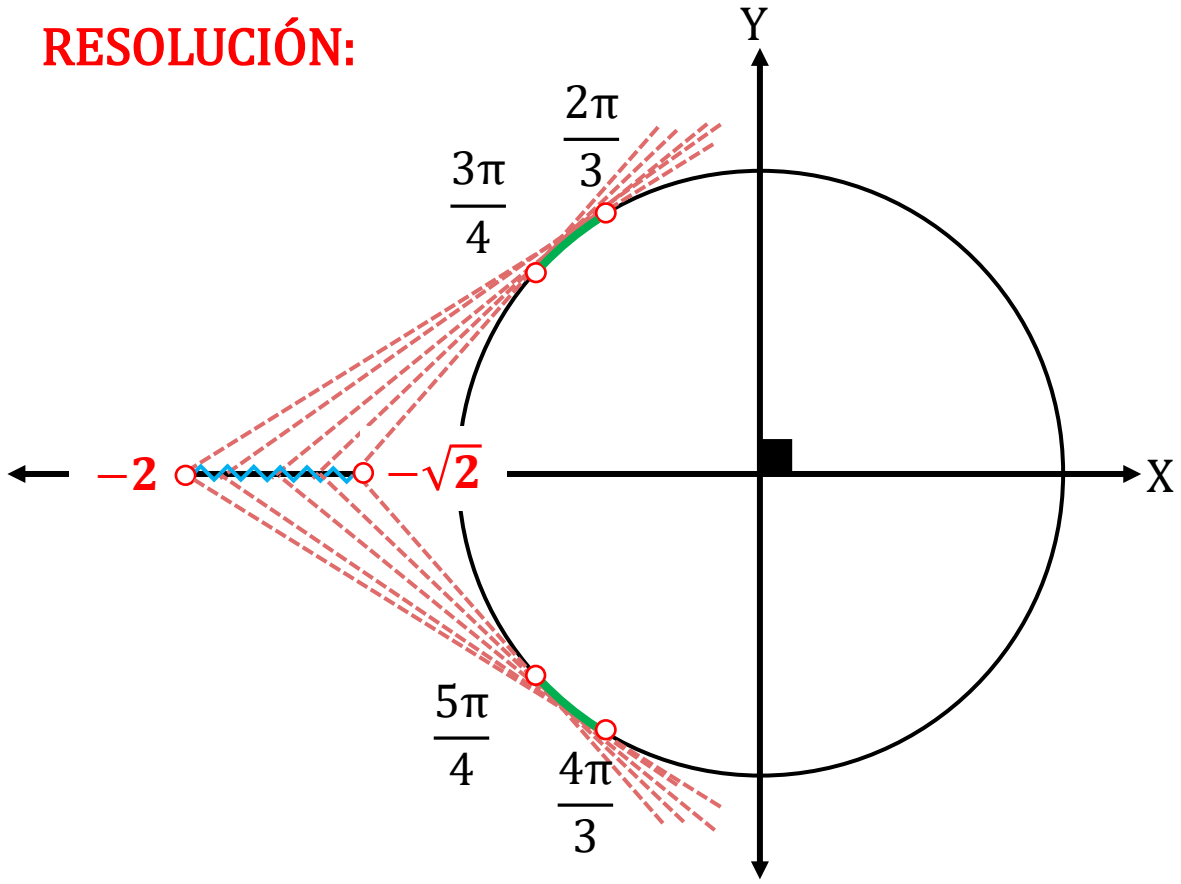
$$\text{Sec}\left(\frac{1}{2}\right) \leq \text{Sec}\theta \leq \text{Sec}1$$

$$\therefore \ \mathbf{Sec\theta_{Máx} = Sec1}$$

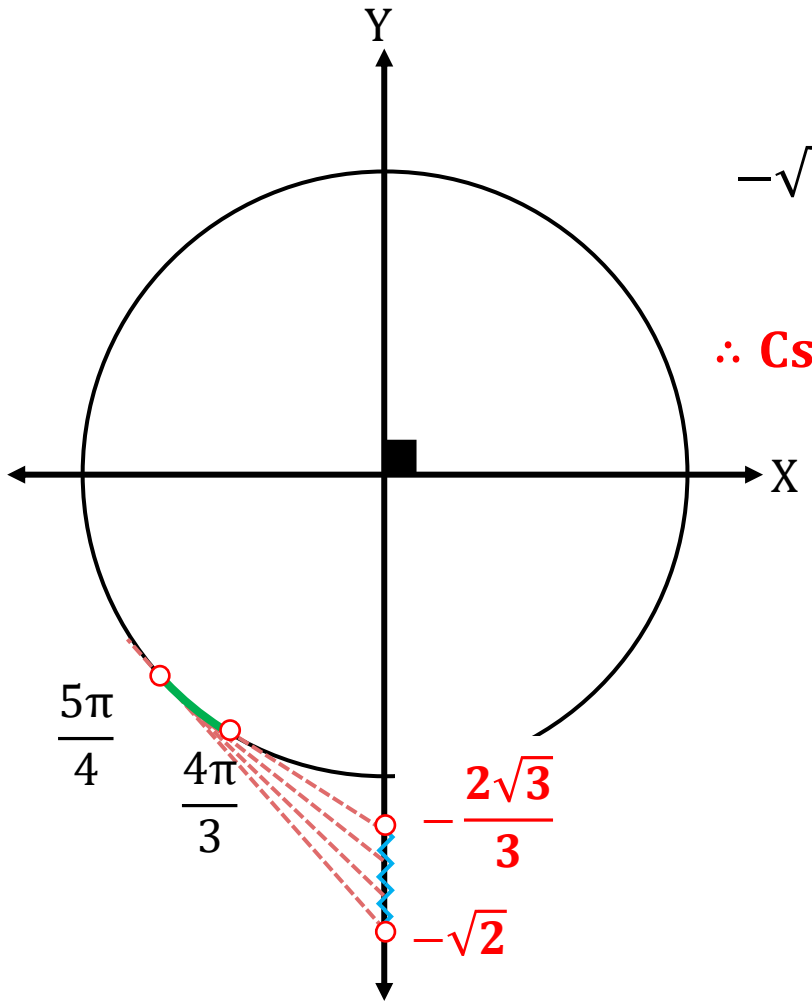
CLAVE: A

14. Si: $-2 < \text{Sec}\varphi < -\sqrt{2}$, hallar la extensión de $\text{Csc}\varphi$, cuando $\pi < \varphi < \frac{3\pi}{2}$

RESOLUCIÓN:



$$\frac{5\pi}{4} < \varphi < \frac{4\pi}{3}$$

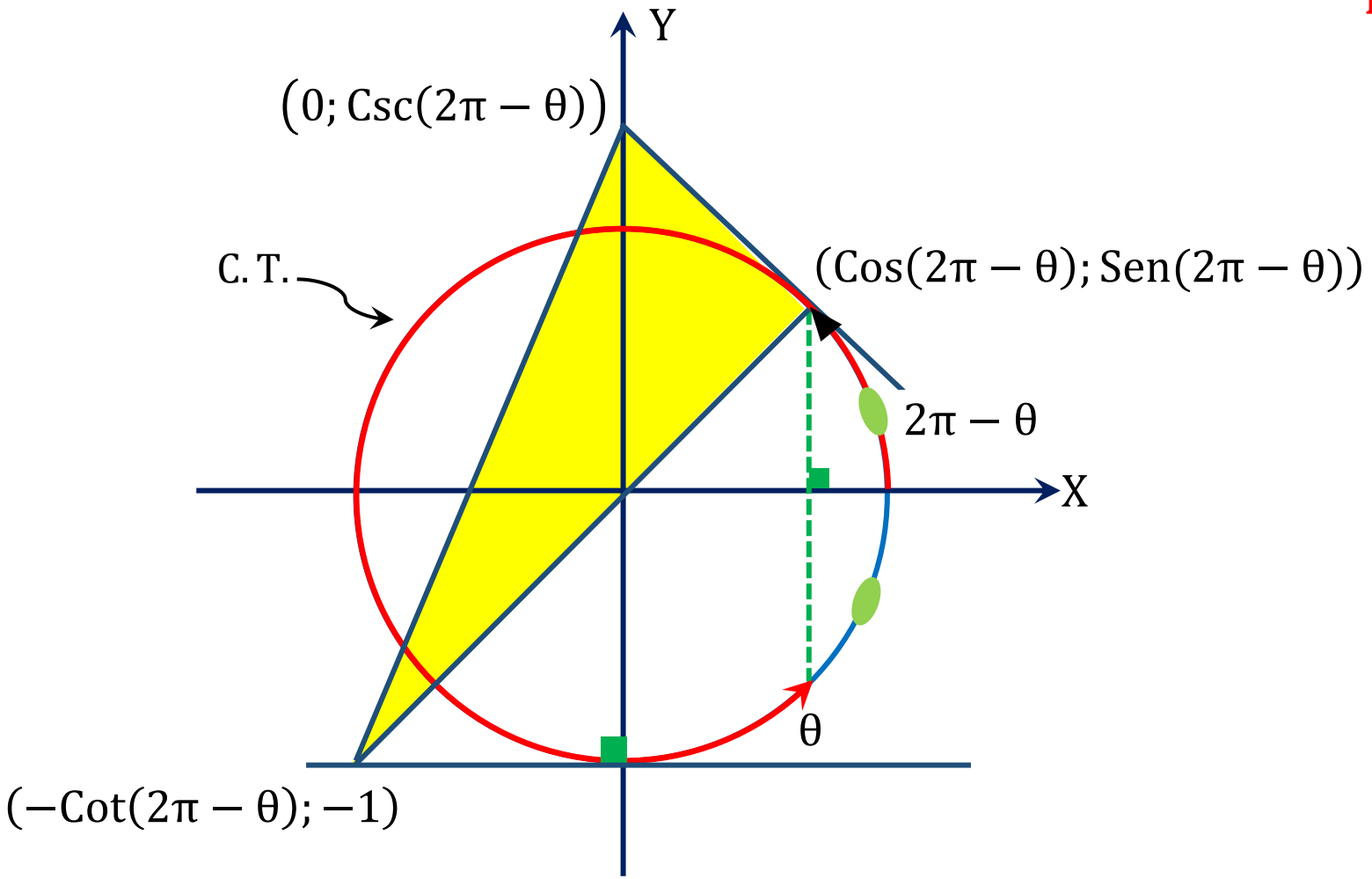


$$-\sqrt{2} < \text{Csc}\varphi < -\frac{2\sqrt{3}}{3}$$

$$\therefore \ \mathbf{Csc\varphi} \in \left\langle -\sqrt{2}; -\frac{2\sqrt{3}}{3} \right\rangle$$

CLAVE: C

15. Hallar el área de la región sombreada



RESOLUCIÓN:

$$\underbrace{\begin{matrix} -\mathrm{Cot}\theta.\mathrm{Csc}\theta \\ -\mathrm{Cos}\theta \\ 0 \end{matrix}}_{I} \left| \begin{matrix} 0 & -\mathrm{Csc}\theta \\ \mathrm{Cot}\theta & -1 \\ \mathrm{Cos}\theta & -\mathrm{Sen}\theta \\ 0 & -\mathrm{Csc}\theta \end{matrix} \right| \underbrace{\begin{matrix} 0 \\ -\mathrm{Cos}\theta \\ -\mathrm{Cot}\theta \end{matrix}}_{D}$$

$$S = \frac{-\mathrm{Cot}\theta + \mathrm{Cot}\theta.\mathrm{Csc}\theta}{2}$$

$$\therefore \mathbf{S} = \frac{\mathbf{Cot\theta(Csc\theta - 1)}}{\mathbf{2}}$$

CLAVE: B

16. Sobre una circunferencia trigonométrica se ubica un arco “θ” en posición estándar, que pertenece al IIC, siendo “P” el extremo del arco “θ”. Hallar la distancia entre el punto “P” y el segmento $\overline{AB'}$ donde $A = (1; 0)$ y $B' = (0; -1)$.

CLAVE: B

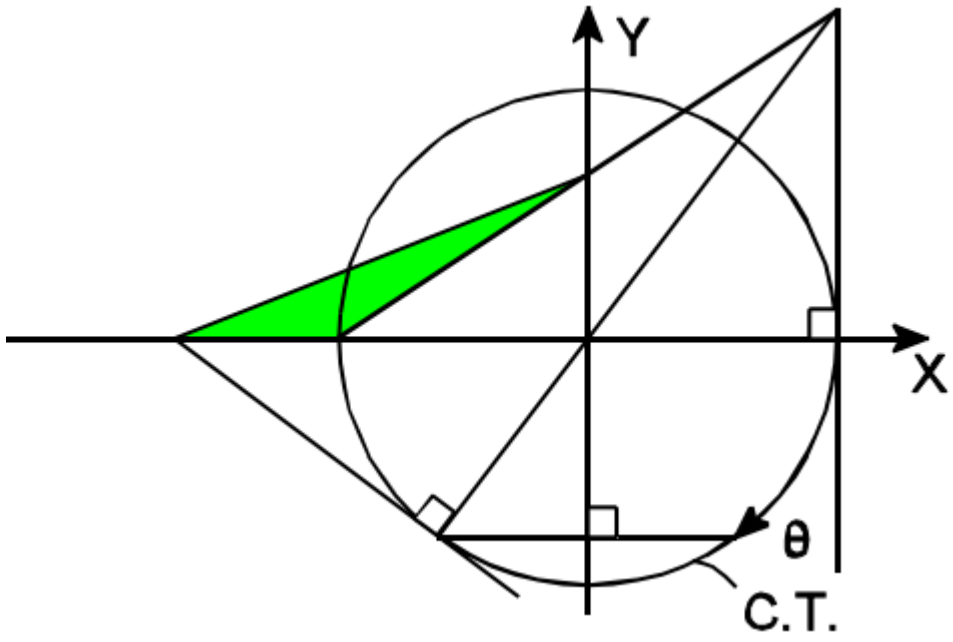
17. Calcular los valores de “x” en el recorrido de $[0;\pi]$, si la expresión: $\text{Senx}.(\text{Versx}) - \text{Cosx}.(\text{Covx})$, adopta valores negativos.

CLAVE: A

18. Si: $\frac{3\pi}{4} < \alpha < \frac{4\pi}{3}$; hallar la extensión de "n", para que se cumpla la igualdad: $\text{ExSec}\alpha = \frac{2+n}{n-3}$

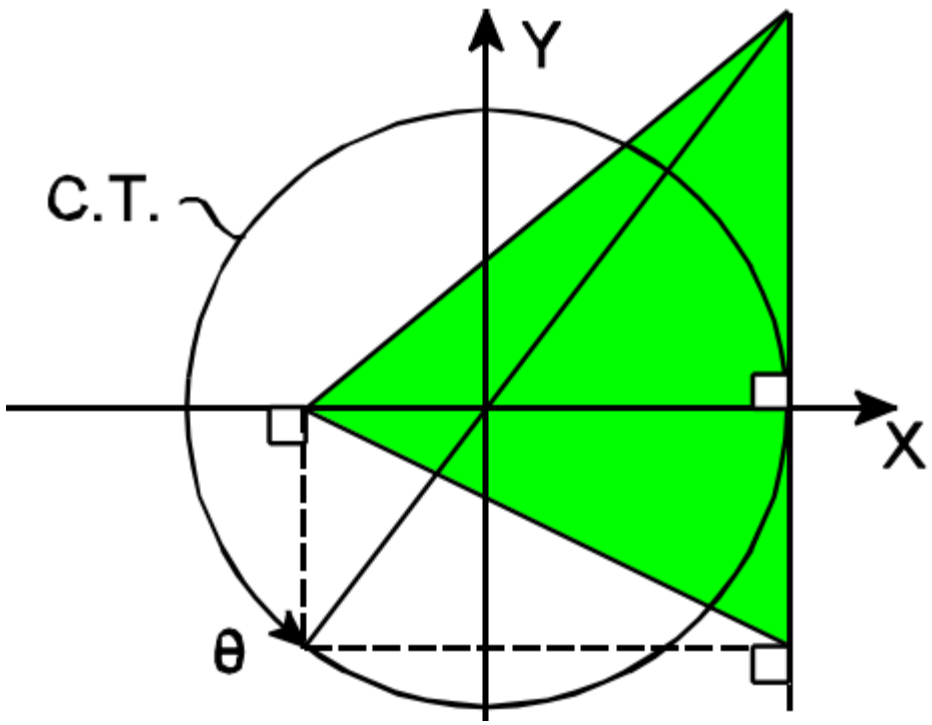
CLAVE: B

19. Hallar el área de la región sombreada:



CLAVE: B

20. Del gráfico, calcular el área de la región sombreada:



CLAVE: D

FIN DE LA SESIÓN

PRACTICA Y APRENDERÁS